Scaffolding Young Math Learners to be Effective Spatial Problem Solvers

Beth Casey
Boston College

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Plan for Presentation: 2 Parts

Part 1: Research Rationale for Scaffolding Spatial Problem Solving Skills in Young Math Learners:

Present our NSF-funded research findings

- What are spatial skills?
- Why are they Important for STEM?
- Why are they particularly important for girls?
- Are early spatial skills important for later math performance in girls?
- Is scaffolding of spatial skills particularly important for girls?

Part 2: Effective Strategies for Scaffolding Early Spatial Skills

2 Video Examples of Spatial Scaffolding:

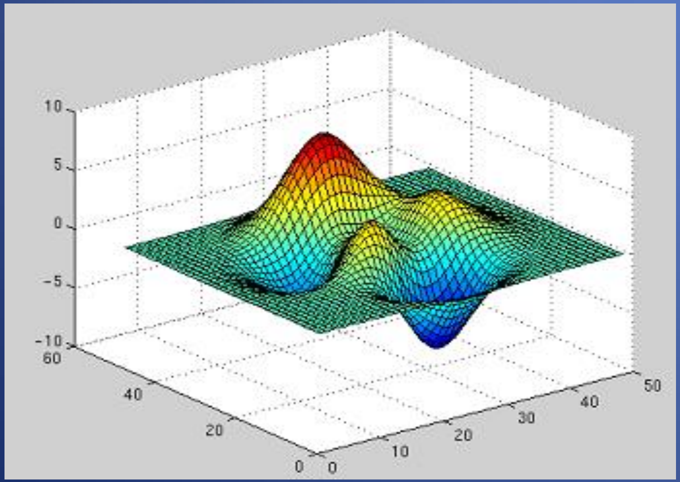
Measurement and Block Building

- Provide information about key developmental changes in measurement and block building during early childhood
- Ask you to identify strategies for scaffolding spatial reasoning used in videos:
  - Use of stories and characters to pose spatial problems
  - Use of materials that pose spatial problems
  - Guiding spatial problem solving by posing questions

Part 1: Research Rationale for Scaffolding Spatial Problem Solving Skills in Young Math Learners:

What are spatial skills?

Spatial skills involve the ability to think and reason through the comparison, manipulation, and transformation of mental pictures.



- Spatial reasoning -- alternate route to solving many math problems – compared to analytical/logical-deductive solutions.
- Spatial strategies can be applied when:
  - using diagrams, and drawings as a method of solving problems
  - considering how fractions can be broken down into geometrical regions
  - estimation and measurement
  - graphing
  - conceptualizing mathematical functions

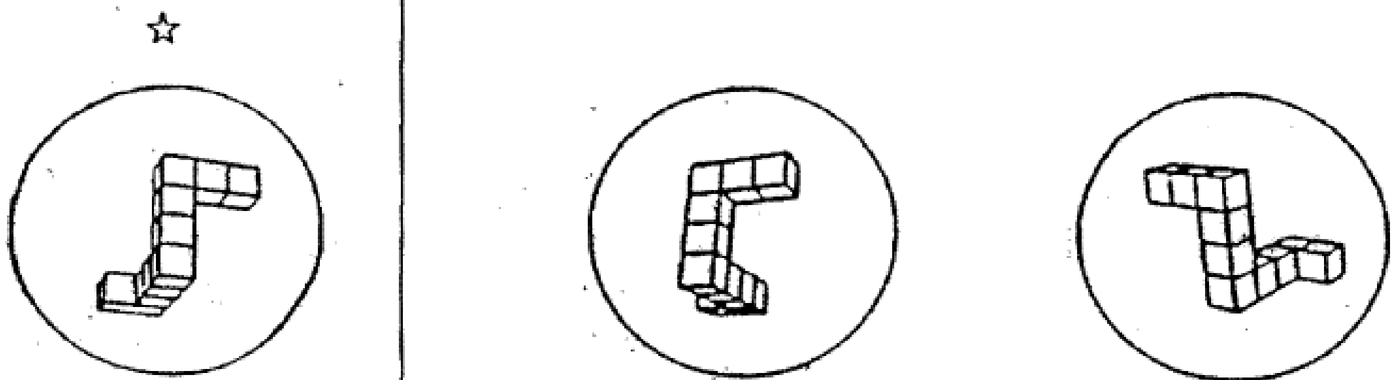
Why are Spatial Skills Important for STEM?

- Longitudinal study (400,000 students) (Wai, Lubinski, & Benbow. 2009)
  - **Findings: Spatial reasoning skills in high school predicts choice of STEM majors and careers**
    - **above and beyond effects of verbal and math abilities**

- The National Science Board (2010) concluded that, “a talent highly valuable for developing STEM excellence -- spatial ability -- is not measured and hence missed.”

Why are spatial skills particularly important for girls?

- The relationship between spatial skills and success in STEM is related to wide range of STEM careers in which women are underrepresented
- Gender differences favoring males in one type of spatial skills (**Skill at 3-D Mental Rotation)** shows the largest cognitive gender difference (Voyer, Voyer, and Bryden, 1995)
- Gender differences in spatial skills start as early as 3 years of age (Levine et al., 1999; Vasilyeva & Bowers, 2006)
- More highly developed and technological societies have wider gender gaps in 3-D Mental Rotation skills than countries with less gender equity and less development (Lippa, Collaers, & Peters, 2010)



Why Did We Study Only Girls?

- Relation between spatial skills and math achievement stronger in girls than boys
- To gain greater understanding of individual variations *within* young girls’ spatial skills
- To understand how spatial skills contribute to math performance in girls over time
- To understand home factors contributing to individual differences in girls

Study 1 Longitudinal Research Questions

- Which of girls' **1st grade skills** will be a stronger predictor of their 5th grade **Math Reasoning Skills**?
  - 1st grade Spatial skills
  - 1st grade Verbal skills
  - 1st grade Arithmetic skills

- Will **1st grade spatial skills** predict:
  - **Only** 5th grade **Math Reasoning Subtest – Spatial**?
  - **Both** 5th grade **Math Reasoning Subtest – Spatial and Analytical**?

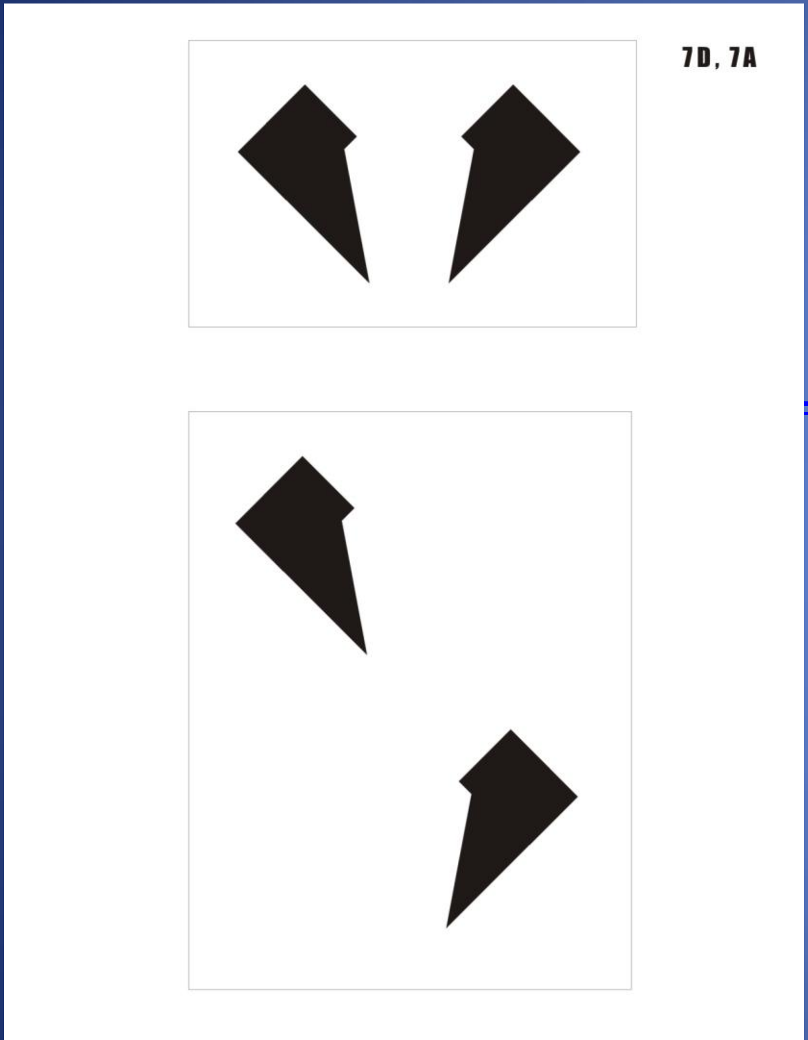
Study 1: Longitudinal Follow-up of 1st Grade Girls into 5th Grade

- **1st Grade** Participants
  - 120 first grade girls from wide range of income levels
- **1st Grade** School Assessments
  - Spatial skills
  - Verbal skills
  - Arithmetic skills

1^{st} Grade Measures: Study 1

- Spatial Measures
  - WISC-IV Block Design Subtest
  - 2-d Mental Rotation Test (Levine et al., 1999)

- Verbal Measure
  - Peabody Picture Vocabulary Test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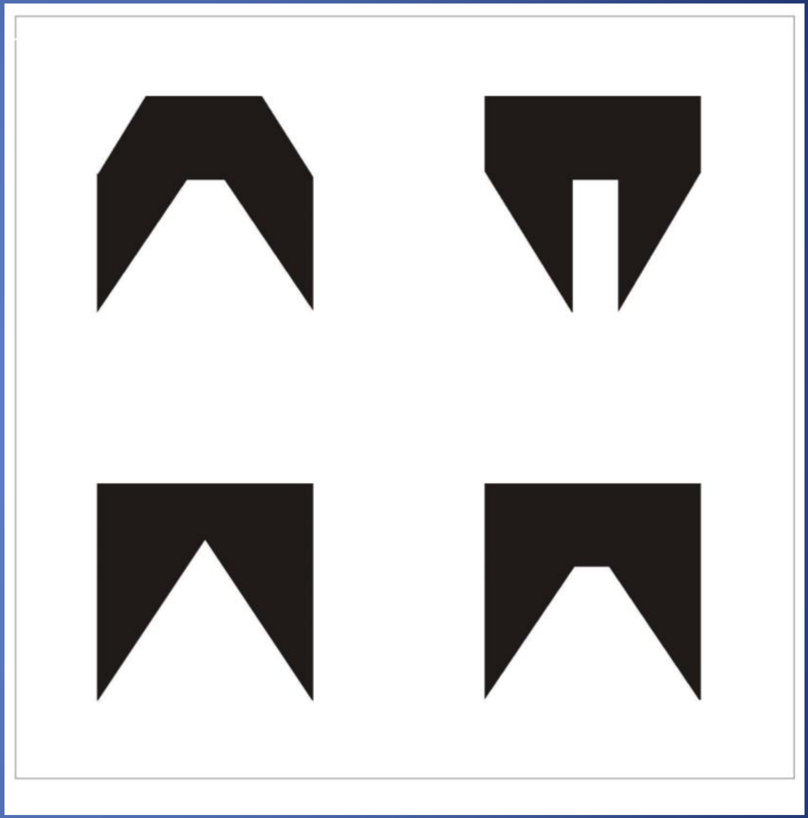

1st Grade School Measures (con’t)

- Arithmetic Measures: Sample Math Problems
  - 10 Addition problems
    - 5+13
    - 18+6
    - 16+9
  - 10 Subtraction problems
    - 17-5
    - 13-8
    - 19-9

Study 1: Longitudinal Follow-up of 1st Graders into 5th Grade

5th Grade Methods

- Participants and Procedures
  - 79 girls from 1st grade study still available in 5th grade
- Classroom assessments: Math measure:
  - Math Reasoning Assessment based on released items from TIMSS, NAEP, MCAS
  - 2 types of math reasoning items
    - **Math Reasoning – Spatial (measurement & geometry)**
    - **Math Reasoning – Analytical (numerical and algebra)**

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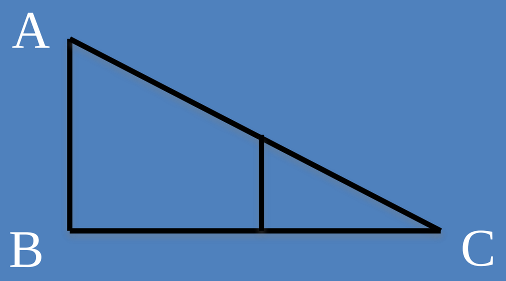
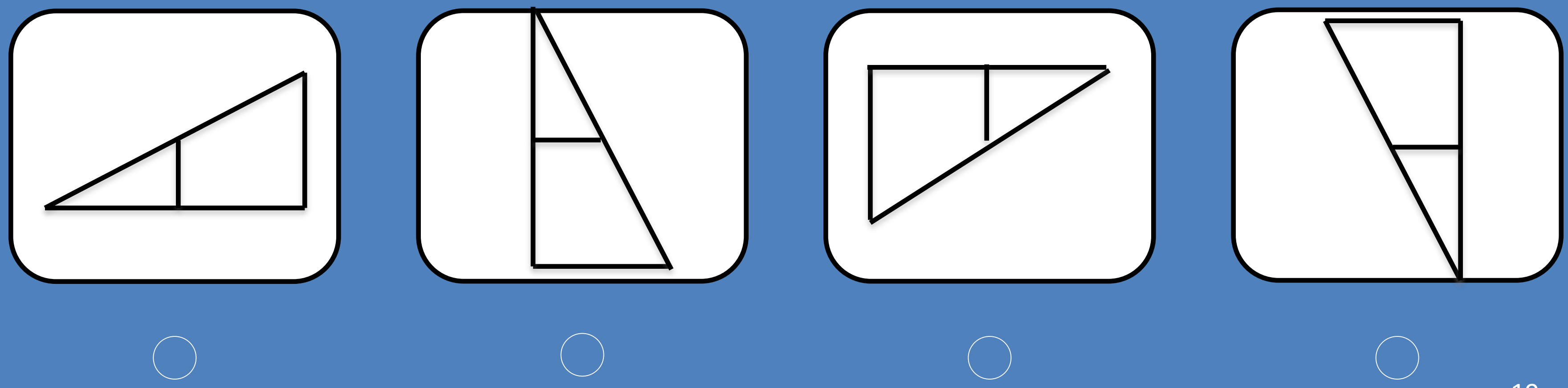


Figure 1



Puppy's Age	Puppy's Weight
1 month	10 lbs.
2 months	15 lbs.
3 months	19 lbs.
4 months	22 lbs.
5 months	?

If the pattern of the puppy's weight gain continues, how many pounds will the puppy weigh at 5 months?

30 lbs.

27 lbs.

25 lbs.

24 lbs.

Table. Standardized Coefficients

		Outcome Variable	
		5th Gr. Math Reasoning Spatial	5th Gr. Math Reasoning Analytical
Predictor Variable		Stand. Coefficient	Stand. Coefficient
1st Grade Spatial			
	Total Effect	0.44	0.37
1st Grade Arithmetic			
	Total Effect	0.18	0.23
1st Grade Verbal			
	Total Effect	0.14	0.12

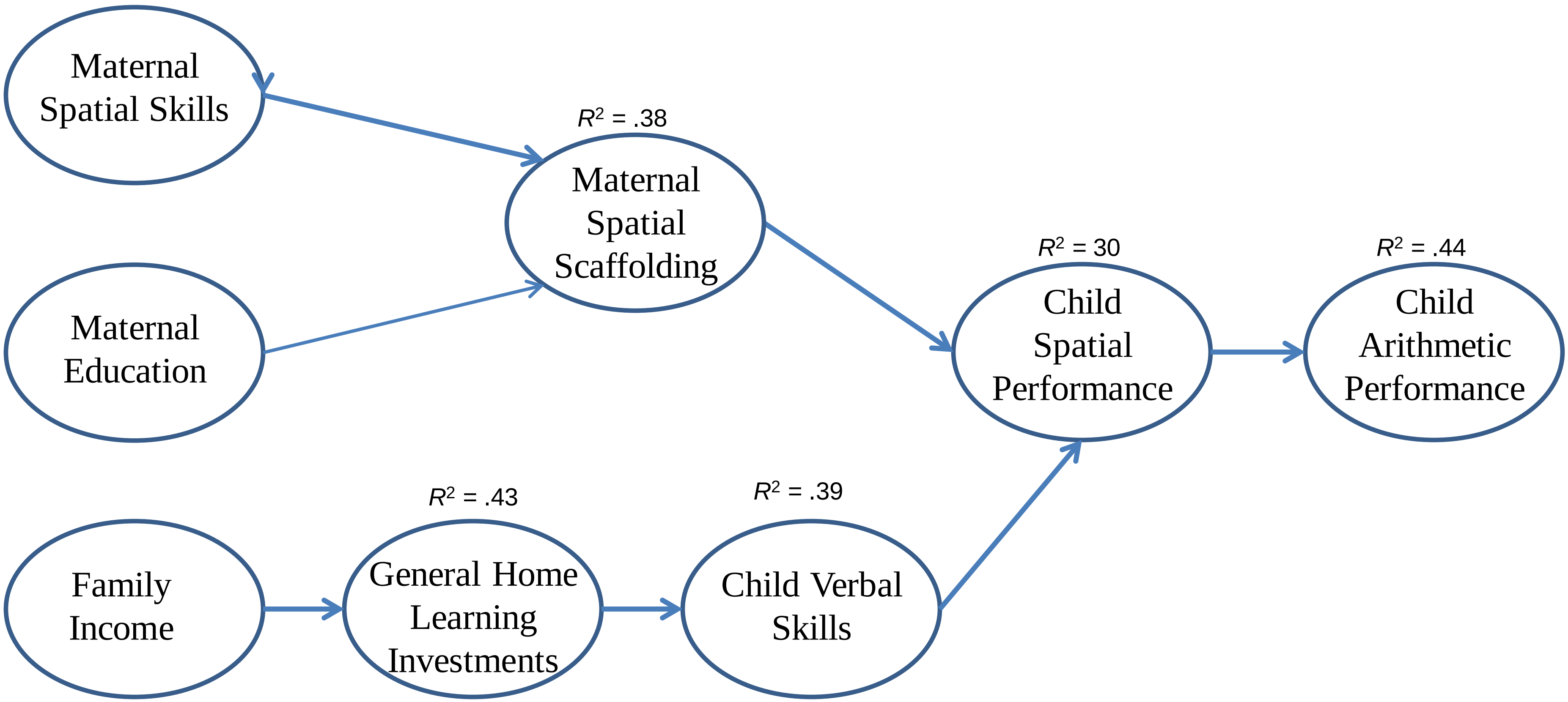
Summary of Findings

- 1st grade spatial skills = stronger predictor of 5th grade math reasoning skills than either 1st grade arithmetic or verbal skills

- Relationship found across **both** types of math reasoning
    - **Math Reasoning – Spatial**
    - **Math Reasoning – Analytical**

Study 2 on 1st Grade Girls: Effects of Home Environment

- What factors in the home environment predict girls' spatial skills?
  - Amount of spatial activities in the home
  - Maternal support of spatial problem solving during a joint maternal-child origami problem solving task
- What are the causal pathways leading from home environments to:
  - Spatial skills?
  - Arithmetic skills?


Maternal Spatial Skills
Maternal Education
Family Income
R^2 = .38
Maternal Spatial Scaffolding
R^2 = 30
Child Spatial Performance
R^2 = .44
Child Arithmetic Performance
R^2 = .43
General Home Learning Investments
R^2 = .39
Child Verbal Skills

Study 2: Summary of 1st Grade Findings

- Simple exposure to spatial activities did not predict spatial skills in girls
- Effective maternal scaffolding of spatial skills did strongly contribute to girls’ spatial skills
- Spatial skills most direct predictor of arithmetic performance

Further Evidence of Importance of Scaffolding for Girls

- 4-year-old girls' puzzle skills were predicted by: (1) amount of maternal spatial support, and (2) maternal use of spatial language during infancy – effect not found for boys (Levine et al., 2012)

- Kindergarten boys improved on puzzle task w/o intervention, while girls improved when scaffolded by school intervention (Casey, Erkut, et al., 2008)

- Research on block building in older students
  - boys and girls interact with blocks in very different ways with boys focusing more on the structural properties of the block building activity (Casey, Pezaris, & Bassi, 2012).

Conclusions

Findings support a converging body of evidence:

Optimal condition for development of young girls' spatial skills = provide some type of support or scaffolding

Simple exposure to spatial experiences not sufficient

School Implications: Most **spatial** activities provided during free play in pre-k-2 grades

Little structured/scaffolded activities –

compared to arithmetic

Part 2 of Presentation

Video Examples of How to Scaffold Spatial Skills in Young Learners

Measurement

Block Building

Strategies for Scaffolding Spatial Skills in Young Learners

- Understand developmental changes in spatial skills of children you are working with
- Use stories and characters to pose spatial problems
- Design materials to pose spatial problems
- Pose guided questions to facilitate spatial problem solving

Rationale for Storytelling

NSF-funded books in 1998 -- now out of print

2 studies showed story/math condition more effective than math w/o story

- Children's mathematical problems arise directly out of math problems encountered by characters in the story.
- Use of animal characters as puppets who pose problems to children
- Benefits girls by providing a relational connection between characters in story and by having students "help" characters
- Benefits boys by having hands-on interactive math within an adventure story context

Rationale for Design of Materials

- Provide open-ended materials so spatial problems can be solved in a variety of ways

- Use materials that pose spatial problems by the very way they are designed and incorporated into the lessons – Examples:
  - Provide materials that require translating of 2-d instructions into 3-d object
  - Provide materials that require producing something a specific size
  - Provide materials that limit the possibility of the easy solution

Rationale and Strategies for Posing Guided Questions

Rationale: Providing open-ended questions enables spatial problems to be solved in a variety of ways

Strategies:

- Ask students to give reasons for their answers
- Point out strategies students suggest and ask other students if that would work
- Rephrase questions when no responses given
- Compare different solutions and test them out
- Have an understanding of students' prior level of knowledge and introduce content for background material when needed

Measurement Video

Story context:

- Frogs in the rainforest have annual frog fiesta competitions
- Ruler of rainforest has decreed they must determine the winners of each context
  - How can you determine which is longest frog when placed around frog fiesta circle?
    - Open-ended materials: string, straws, pipe cleaners, strips of paper, etc.
  - How can you use the large units of measure (Grumpy frogs) and the small units of measure (Froglets) to measure length of frog jumps?
  - Which type of frog can hop the furthest in a set time using a ruler to measure?

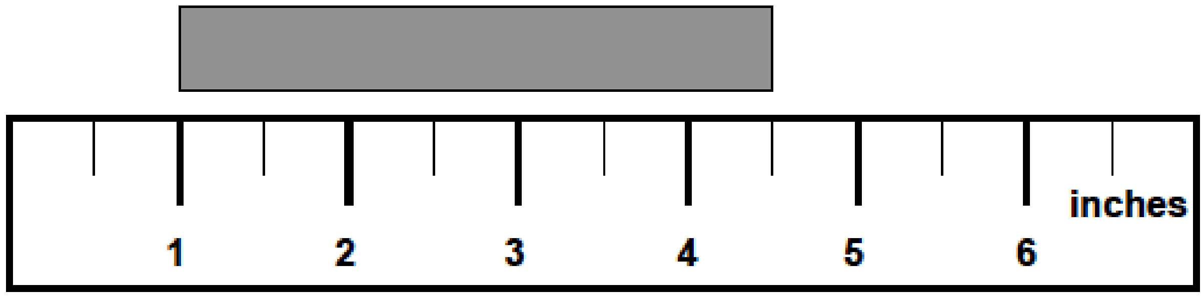
Developmental Changes in Measurement Reasoning in Young Learners

- Children have to develop “measurement sense”
  - To understand the purpose and reason behind measurement

- Instead children learn the measurement procedures by rote
    - Ex: if ruler is not aligned with pencil being measured, children just report # on ruler where pencil ends.

6

How many inches long is this stick?



5 inches

$4\frac{1}{2}$ inches

7 inches

$3\frac{1}{2}$ inches

Young Learners:
Areas of Confusion When Measuring (con't)

Confusions with Number:

•To measure length one must know that arrangement and spacing matter

- Transferring from numerical reasoning, students often count the numbers on a ruler rather than the spaces between

Young Learners:
Areas of Confusion When Measuring (con’t)

Confusions about Placement :

•To measure with any unit, child needs to understand that at the start, measurements of each object must begin at same baseline

•To measure with any unit, whether standard or non-standard, the units must be placed end to end, with no gaps or overlaps

Young Learners:
Areas of Confusion When Measuring (con’t)

Confusions about Size of Unit:

•Transition from non-standard units to standard units: recognize that a 1” non-standard unit = the spaces <u>between the lines</u> on a 12” ruler

•To measure an object’s length, one must use the same size unit of measure for the entire process of measuring it

•When using a larger unit of measure, fewer units needed to cover space than with smaller unit of measure – The Inverse Rule

–confusion with numbers – larger means more

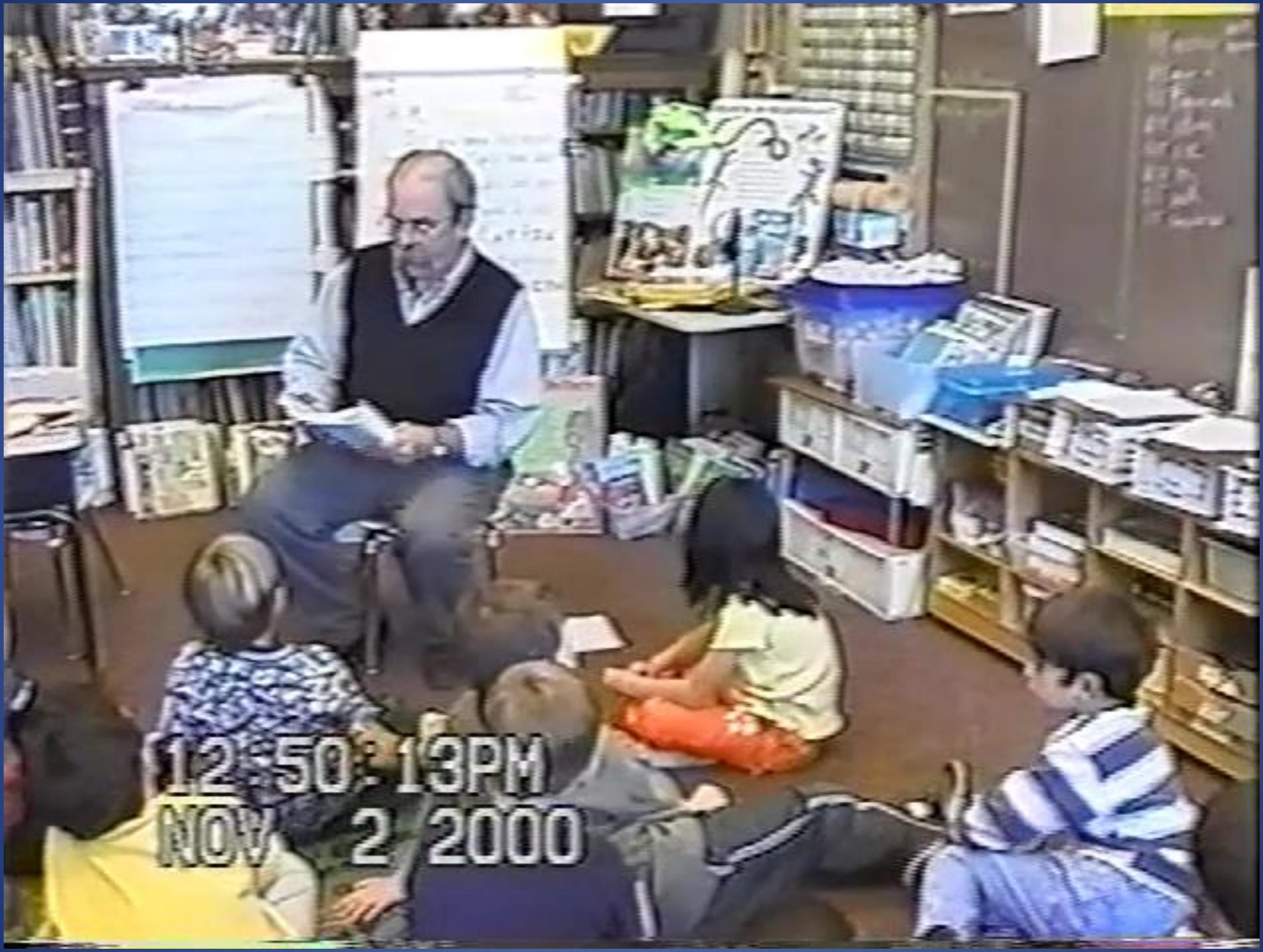
Measurement video

How is Spatial Problem Solving Scaffolded by 1st Grade Teacher?

Use of story

Use of materials

Use of questions and gestures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Block Building Video

Story context:

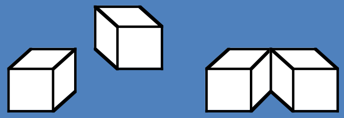
- Sneeze the Dragon takes the kindergartners back in time to the middle ages

- The King and Queen want to build a castle for their families and they ask Sneeze to help build models of the castles -- Sneeze knows that her kindergartners can help her because they know how to build with blocks

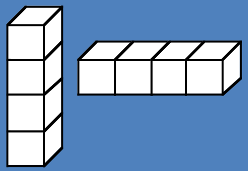
- The King and Queen have requirements for their castle based on developmental changes in block building

Developmental Ratings of Block Building

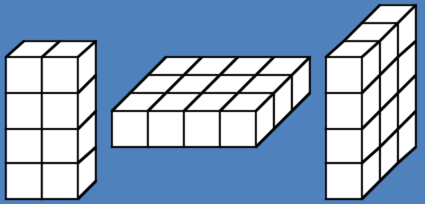
Level 0: Random block placement



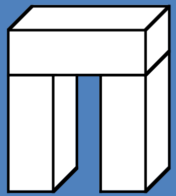
Level 1: 1-d (1-dimensional) structures – row of single blocks, or stack of single blocks – 2 years of age



Level 2: 2-d structures (no internal space) – structure with no width (a wall), no height (a floor), or no length (a 2 block-wide tower) -- 2-3 years

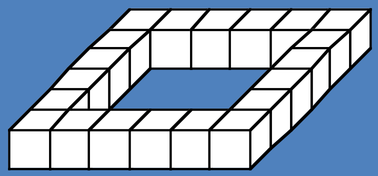


Level 3: 2-d structures with vertical internal space – arches -- 2-3 years

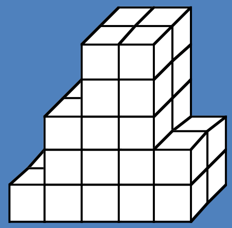


Developmental Ratings of Block Building (con't)

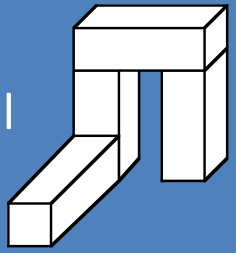
Level 4: 2-d structures with horizontal internal space –enclosure only 1 block high (no height) – 3 years of age



Level 5: 3-d structures – 3-d piles with no internal space 3 years of age

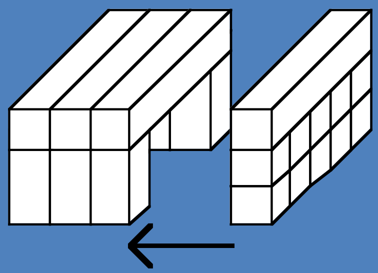


Level 6: 3-d structures – 2-d vertical or horizontal internal space plus depth to make 3-d structure – 4 years

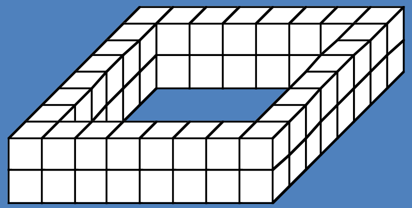


Developmental Ratings of Block Building (con't)

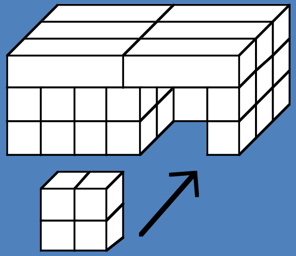
Level 7: 3-d horizontal enclosure: 1 block-high enclosure + layer of roof blocks – adds height to make a 3-d structure -- 4-5 years of age



Level 8: 3-d horizontal enclosure 2 blocks-high – 4-5 years of age



Level 9: 3-d horizontal enclosure, 2 blocks high + roof + divided internal space – 5-7 years of age



What are the requirements for the castle and why?

- The walls surrounding the castle have to be at least 2 blocks high so the animals can't escape

- There needs to be a doorway so the King and Queen can go into the castle

- The ceiling/roof has to be a certain size

- The castle walls need to have 3 floors built one upon one another

- There needs to be stairs so the King and Queen can climb up to the top of the tower

Block Building Video
How Was Spatial Problem Solving
Scaffolded by Kindergaarten Teacher?

Use of story

Use of materials

Use of questions and gestures


PLAY
VIDEO CALIBRATION

FUTURE DIRECTIONS RELATED TO EARLY CHILDHOOD SPATIAL MATH

Introduce spatial skills as an integral part of early childhood mathematics

Teach spatial skills to children embedded within in a simple story context.

Pose open-ended spatial problems

Develop spatial concepts systematically through sequenced mathematics problems extended over time (not one-shot lessons)

FUTURE DIRECTIONS RELATING TO BLOCK BUILDING

In addition to unstructured exploratory block play

Teach systematic block building skills:

Base teaching upon mathematical and spatial objectives – following developmental changes

Build block-building skills through sequenced activities

Make sure girls as well as boys learn about structural and spatial aspects of block building