RIMSE: Research Innovations in Mathematics and Science Education

Pathways to Calculus: An Adaptive Model for Transforming Precalculus Level Mathematics Teaching and Learning

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Goals for This Session

- Overview of an approach that is working to transform secondary math teaching and learning
- An illustration of the complexity of the problems that must be overcome to realize change
- An illustration of a process that will enable ongoing improvements and sustained learning
- An invitation to others to collaborate in our work

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What is Precalculus Level Mathematics so Important?

- Foundational for learning calculus and gateway to engineering and sciences

- A requirement for business, nursing, psychology, etc. majors

- Introduces students to modeling in the sciences and engineering

Precalculus: A Major Problem in Generateing STEM Thinkers

-Precalculus Level Curriculum and Instruction is not preparing students to pass calculus or understand key ideas of calculus (e.g., Carlson, 2002; Thompson, 2010; Oehrtman, 2009; Dubinsky, 1992; Zandieh, 2001)

-Far too many high performing university and secondary students are electing to discontinue their study of mathematics after taking precalculus, shutting doors to STEM fields

-Students believe mathematics is about memorizing facts, carrying out procedures to get answers (e.g., Thompson, 2008)

Equipping Teachers to Engage Students in activities that lead to students acquiring:

- Understanding of foundational ideas
- Mathematical practices that have been documented to be essential for solving novel problems

(Common Core State Standards for Mathematics)

Issues that must be addressed

- Assessment/teacher rewards
- Administrator values
- Teacher knowledge/conceptions of key ideas
- Teacher beliefs
- Curriculum
- Current teaching practices

Project Pathways: Working Toward a Solution

- Supporting teachers in shifting their knowledge, beliefs and classroom practices
  - Developing coherent and meaningful curriculum and tools to support teachers in making the shifts
  - Ongoing workshops
  - Providing meaningful assessment
- Engaging school administrators in ongoing dialogue to overcome local challenges

Scaling the *Pathways Professional Development Model* for Teaching Secondary Mathematics

-Workshops/graduate courses focused on developing mathematical content knowledge for teaching key ideas and reasoning abilities of algebra through precalculus

-PLC's Professional Learning Communities that promote quality reflection on student thinking relative to their teaching

-Curriculum, Assessments, and Teacher Support Tools Student investigations; conceptually oriented assessments, online videos, annimations. (Carlson & Oehrtman)

Precalculus: Pathways:
Our inquiry based approach

One Example

Design (Revise)

Practices & Instruments

Carlson & Bloom Problem solving framework

-Describes effective mathematical practices

Conceptual Frameworks (e.g., Covariation, FTC, Function)

-Characterizes understanding (e.g., reasoning abilities, connections, notational issues)

Theoretical grounding for

-Designing curricular modules

-Determining course structure

-Instrument development

Lens for researching emerging practices and understandings

Revise Cognitive Frameworks

An Example: Teaching Proportional Reasoning for Understanding (Cross Multiply)

Solve for x:

$$\frac{4}{7} = \frac{15}{x - 3}$$

Foundational Ideas and Reasoning Abilities for Learning Calculus and Modeling in Science

- Covariation and Quantification
- Proportionality
- Function (Composition and Inverse)
- Linear Functions
- Exponential Functions
- Polynomial Functions
- Rational Functions
- Trigonometric Functions (Precalculus)

Quantitative Reasoning

Covariation

Function

(Average) Rate of Change

Laying the Foundation for Success in Calculus (and Beyond)

- Student investigations designed to support their development of foundational reasoning abilities and understandings

- Teacher Support Tools Help Teachers Acquire “Content Knowledge for Teaching”
  - PowerPoint Slides Help Novice Teachers Engage Students in Conceptual Conversations
  - Worksheets (with detailed solutions) help teachers promote critical reasoning abilities and understandings in students

<See PD Tools>

SAT Item

- A total of 120,000 votes were cast for 2 opposing candidates, Garcia and Pérez. If Garcia won by a ratio of 5 to 3, what was the number of votes cast for Pérez?
- (A) 15,000
- (B) 30,000
- (C) 45,000
- (D) 75,000
- (E) 80,000

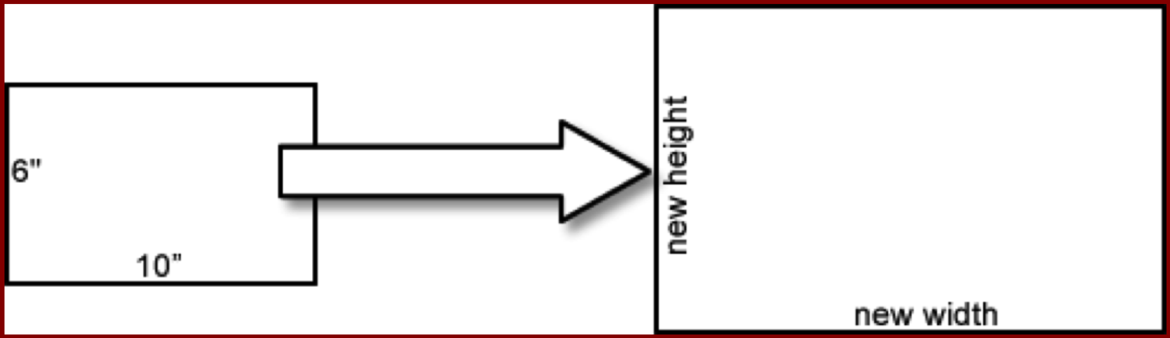
x = # of votes for Perez

$120{,}000 - x$ = # of votes for Garcia

$$\frac{5}{3} = \frac{120{,}000 - x}{x}$$

PHOTO ENLARGEMENT TASK

1. A photographer has an original photo that is 6 inches high and 10 inches wide and wants to make different-sized copies of the photos so that new photos are not distorted.



   a. If the photographer wants to enlarge the original photo so that the new photo has a width of 25 inches, what will the height of the new photo need to be so the image is not distorted? Explain the reasoning you used to determine your answer.

TWO VARYING QUANTITIES ARE RELATED BY A CONSTANT RATIO

Let h = the height of the new photo (in inches)

Let w = the width of the new photo (in inches)

Then, as h and w vary together, their ratio stays fixed

$$\frac{w}{h} = \frac{10}{6} = \frac{5}{3} = \frac{25}{x}$$

$$x = 15$$

CONSTANT MULTIPLE & SCALING

h is always the same multiple of *w*.

- $h_1=6''$ is 6/10 as long as $w_1=10''$

Why?

- So h_2 should be 6/10 as long as $w_2=25''$
- $h_2 = (6/10)\cdot 25''$

If we scale *w* by some factor, we should also scale *h* by the same factor.

- $w_2=25''$ is 25/10 as long as $w_1=10''$

Why?

- So h_2 should be 25/10 as long as $h_1=6''$
- $w_2=(25/10)\cdot 6''$

Pathways Resources for teaching Precalculus, Algebra II, College Algebra & Algebra I

Rational Reasoning:

https://www.rationalreasoning.net/

Classroom Norms (Rules of Engagement) for Supporting Student Learning

The teacher expects (and acts on the expectation that) students:

- Persist in making sense/constructing meaning
- Students are expected to express their thinking and *speak with meaning* about their problem solutions
- Base conjectures on a logical foundation (Integrity)
- Make sense of the meanings conveyed by others/Pose meaningful questions when they don’t understand.

Covariational Reasoning: A foundational way of thinking for modeling in Science and understanding key ideas in calculus

Covariational reasoning:

Imagining how the values of two quantities change together.

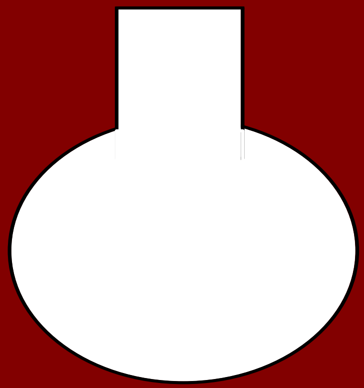
(Thompson, 1994; Carlson et al. 2002)

Covariational Reasoning

- MA1: Imaging 2 quantities that are changing together
- MA2: Imaging how the direction of the 2 quantities are changing together
- MA3: Imaging *how* one quantity is changing while imagining changes in the other quantity
- MA4: Imaging *how* the average rate of change of the output quantity with respect to the input quantity is changing while imaging uniform increments of change in the input quantity.

The Bottle Problem

Imagine this bottle filling with water. Sketch a graph of the water's height as a function of the amount of water that's in the bottle. Explain the thinking you used to construct your graph.



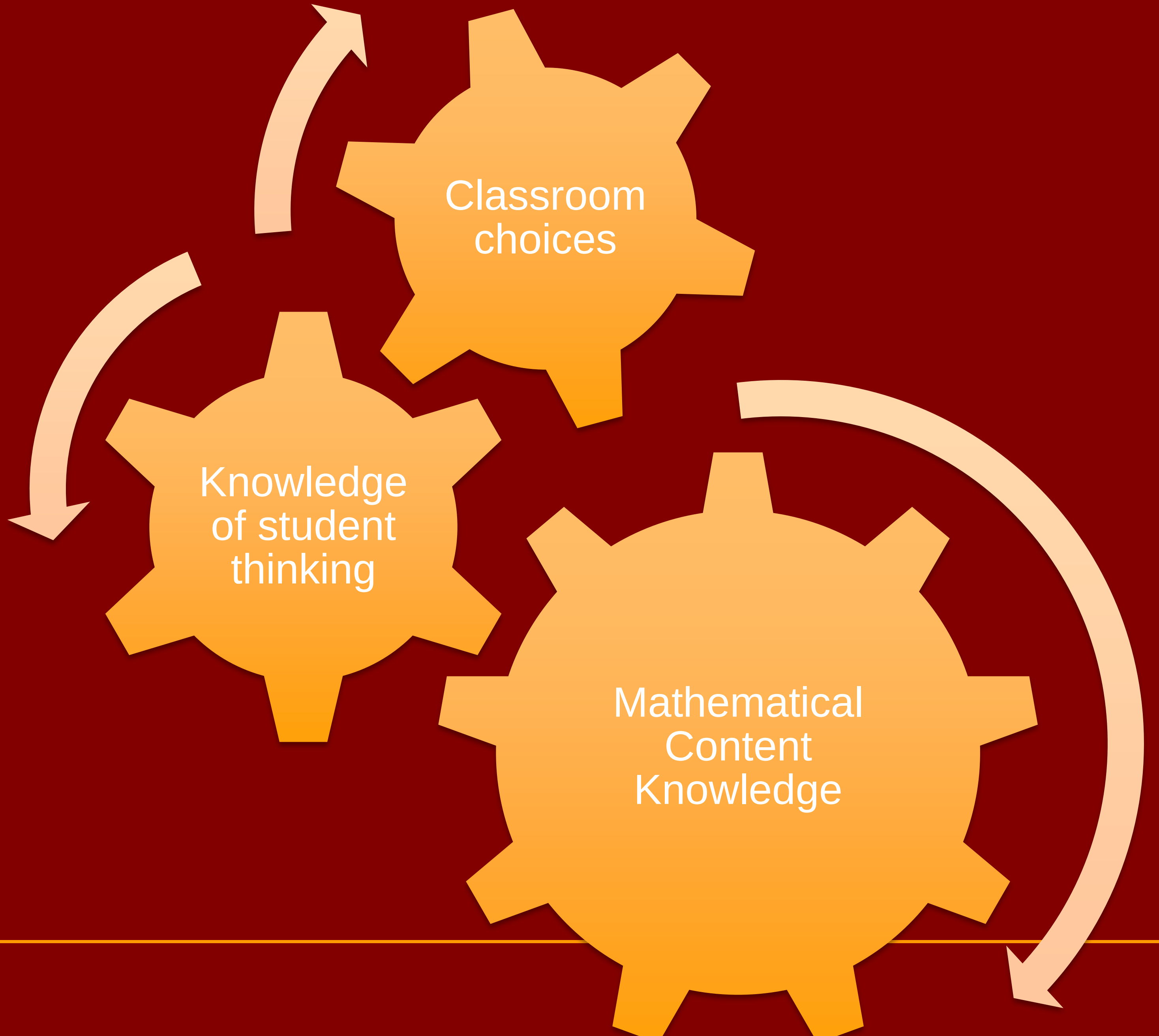
What do we mean by effective teaching?
Teachers who can act effectively to support student learning.

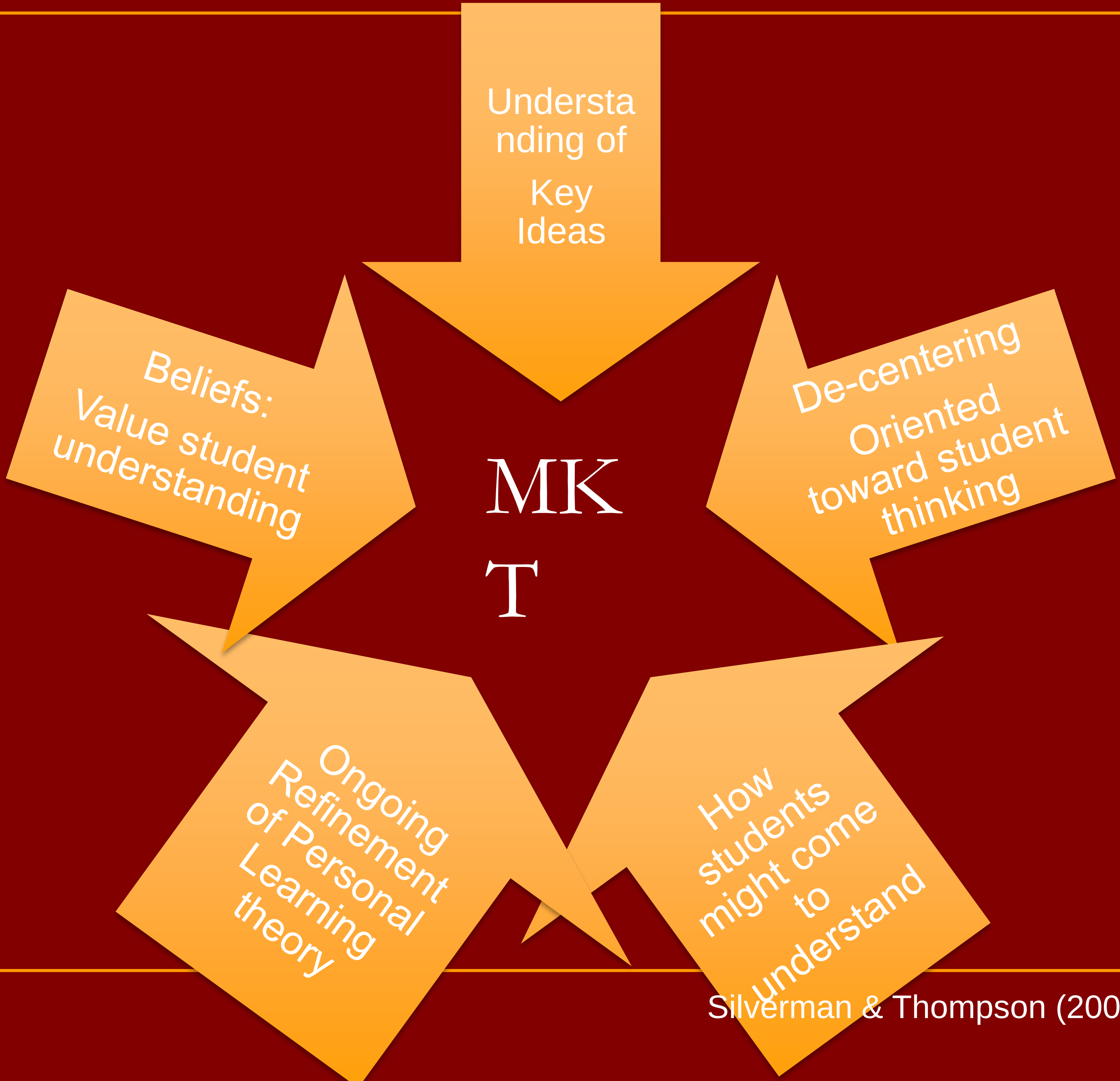
-Understand student thinking, challenge student thinking and support the development of student thinking

Mathematical Knowledge forTeaching (MKT)

Understandings that allow teachers to spontaneously act in ways to support student learning (reasoning, understanding, connections)

Disconnected facts & procedures will not be leveraged to solve problems!!!


Classroom choices
Knowledge of student thinking
Mathematical Content Knowledge



Silverman & Thompson (2008)

The Precalculus Concept Assessment Instrument

- A 25 item multiple choice exam (80 Item Pool)
- Based on the body of literature related to knowing and learning the concept of function (Carlson, 1998, 1999, 2003; Dubinsky et al., 1992; Monk,1992; Thompson and Thompson, 1992; Thompson, 1994; Sierpinska, 1992)
- Validated and refined over a 12 year period

The Precalculus Concept Assessment Instrument (PCA)

- A broad taxonomy identifies specific concepts and what is involved in understanding those concepts
  - Based on broad body of research on knowing and learning those concepts
- Multiple items assess each taxonomy attribute
- Items have multiple choice responses
- Item choices are based on common responses that have been identified in students (clinical interviews)
- Videos of student reasoning are captured for each item choice

Carlson, Oehrtman & Engelke, 2010; Cognition and Instruction

Student PCA Performance

- PCA administered to 550 college algebra and 379 pre-calculus students at a large southwestern university
- Also administered to 267 pre-calculus students at a nearby community college

- Mean score for college algebra: 6.8/25

- Mean score for pre-calculus: 9.1/25

Predictive Potential of PCA

- How does PCA score relate to course grade in calculus?
  - Tracking 277 students in beginning calculus, 87% of students who scored 13 or above on PCA at the beginning of the semester received an A, B, or C in calculus.
  - 85% of the 277 students who scores 11 or below received a D, F or withdrew.

Precalculus: Pathways to Calculus

Summary of PCA pre- and post-test

College Algebra

	Pre-test Mean	Post-test Mean
Fall 09	5.7	11.2

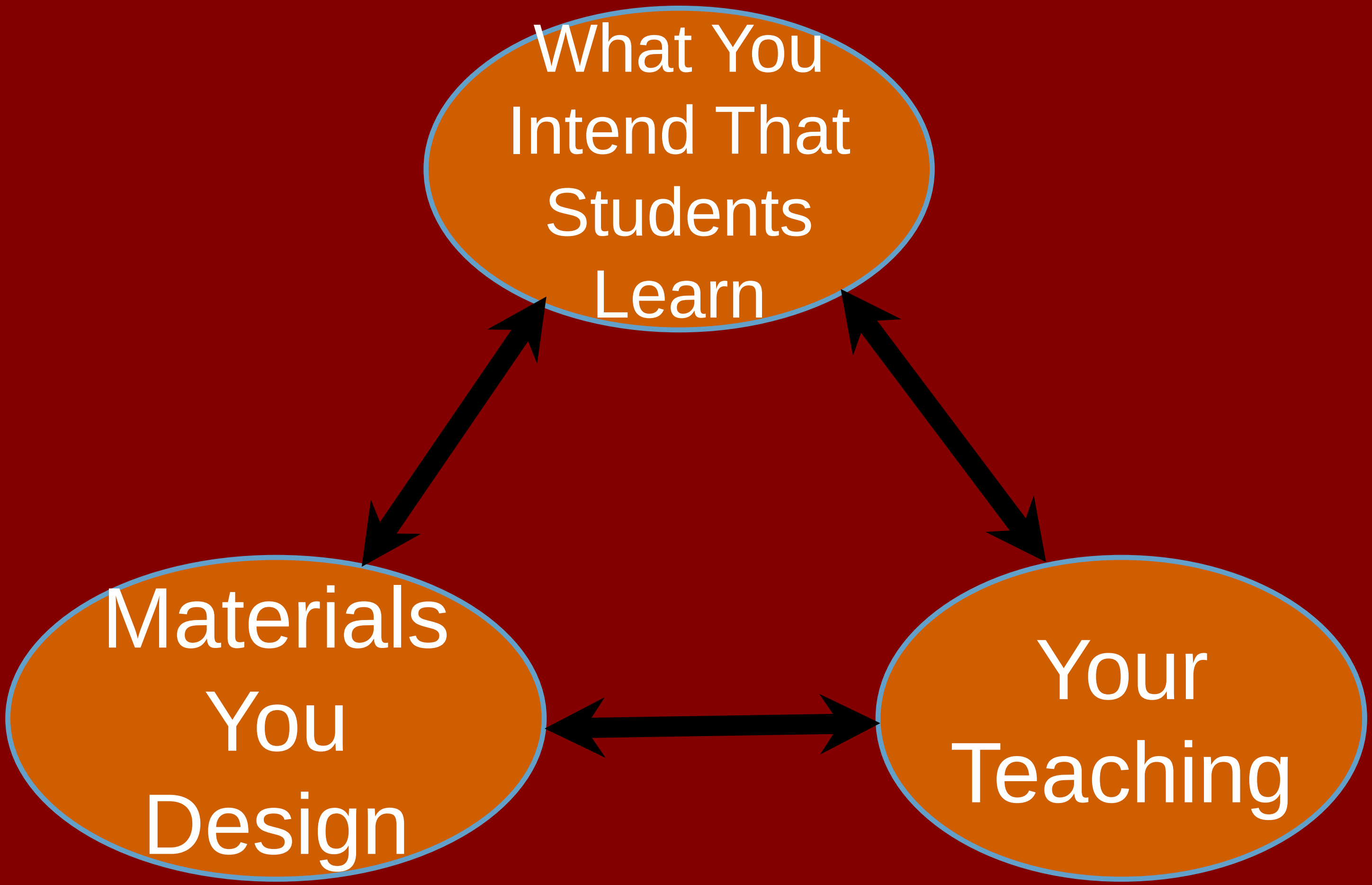
(Previous Best Post Mean Score: 6.8)

Precalculus

	Pre-test Mean	Post-test Mean
Fall 09	8.2	15.2

(Previous Best Post Mean Score: 9.2)

The Didactic Triad



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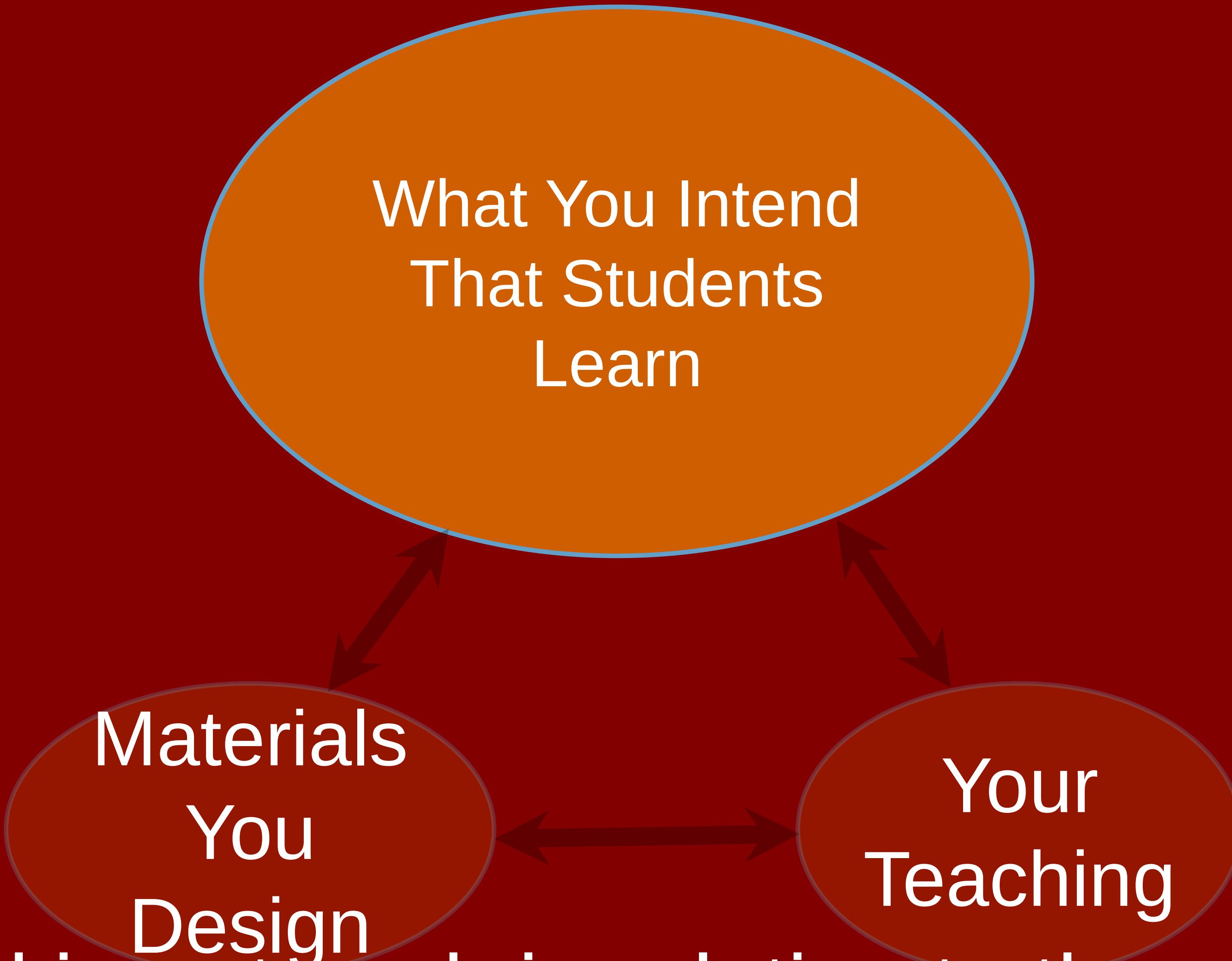
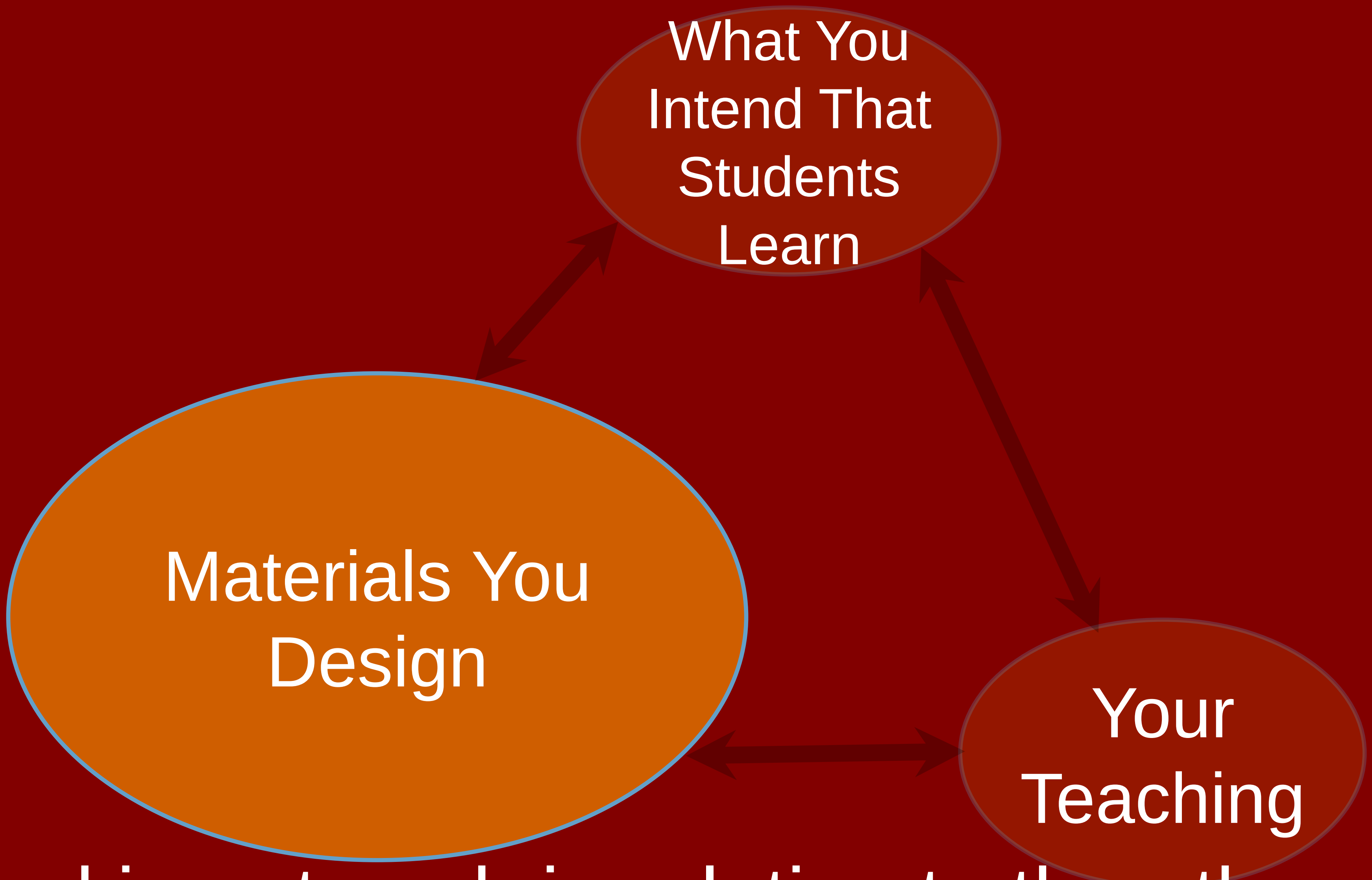


Looking at each in relation to the others

The Didactic Triad



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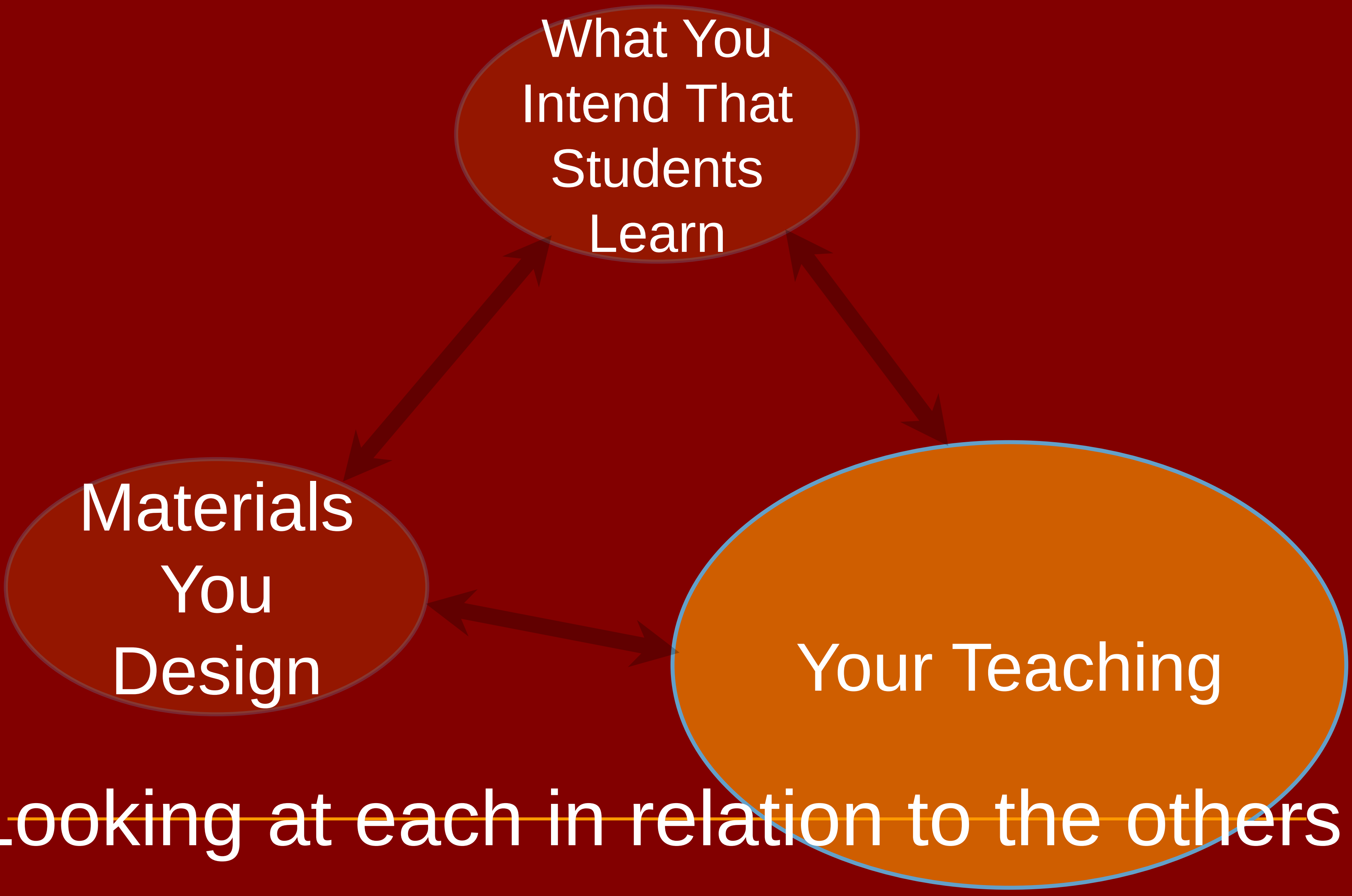

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