Using Learning Trajectories to Unpack and Interpret the Common Core Math Standards

NSF STEM Smart Conference: Lessons Learned from Successful Schools.

University of Nevada, Las Vegas

Breakout Session

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Context—Then and Now

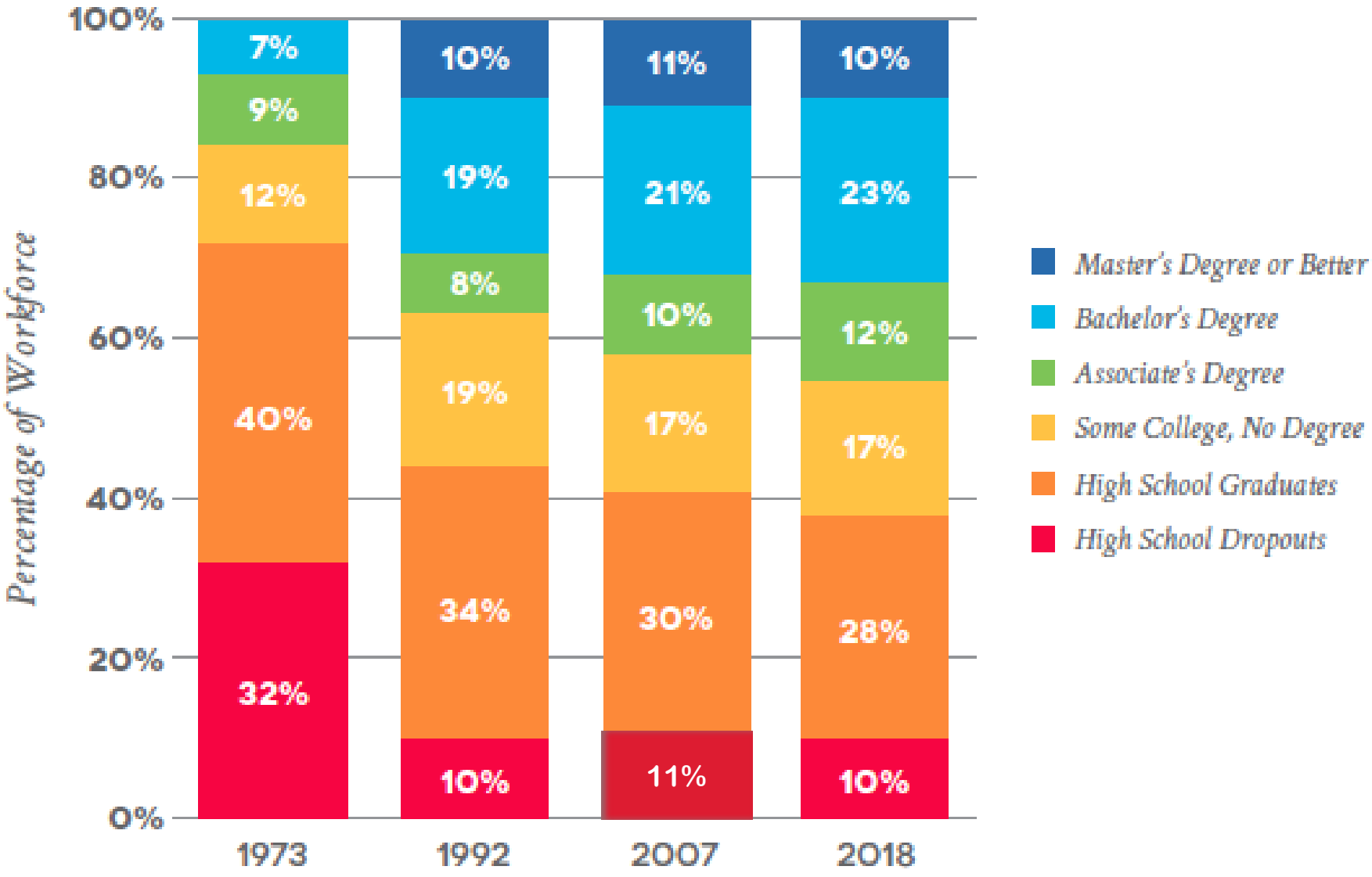
Then

- *NCTM Standards (1989) and AAAS and NRC (Science)*
- *Standards state-by-state*
- *Beginning of NCLB*
- *Disaggregation of data*
- *Nascent technology applications, graphing calculators*
- *Early internet, minimal access in schools*
- *Increasingly mobile student and teacher populations*

Now

- *Common Core State Standards, 45 states*
- *New designs for ESEA needed*
- *Wireless networking and Cloud Computing (shared services model)*
- *Social Networking everywhere*
- *Data-intensive empirical research changing the way science is done, and different demands on quantitative modeling and literacy.*
- *Analytics*
- *Knowledge Intensive Industries and STEM*
- *Increasingly mobile student and teacher populations*
- *Increasing gaps associated with race and poverty*

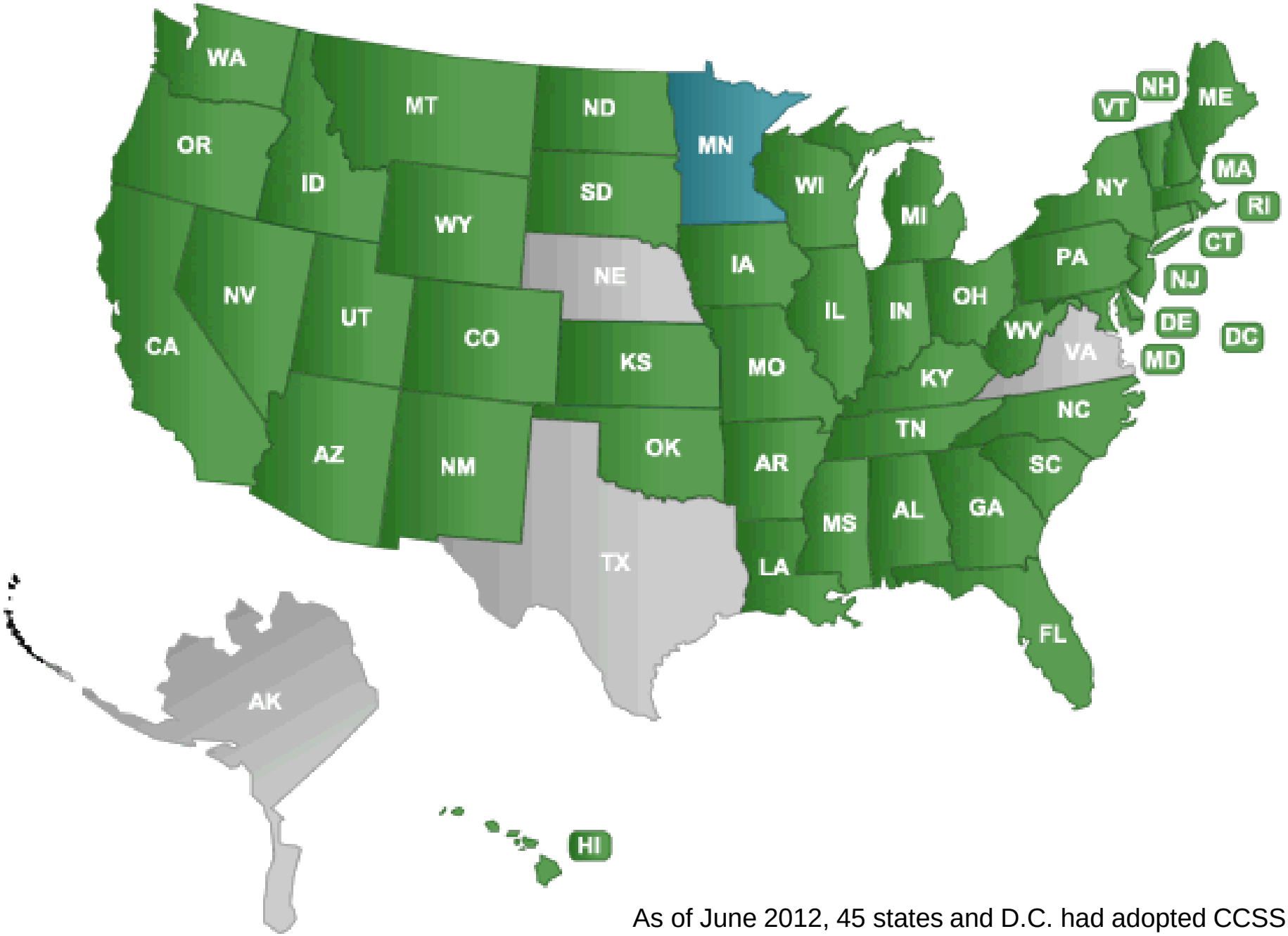
PERCENTAGE OF WORKFORCE BY EDUCATIONAL LEVEL
1973 THROUGH 2018



Source: The Georgetown University Center on Education and the Workforce, 2010.

Brief History of CCSS Development

- July 2009: The development of the College and Career Ready Standards draft, outlining topic areas
- October 2009: Public release of the College and Career Ready Standards
- January 2010: Public release of Draft 1 to states
- March 2010: Public release of Draft 2
- June 2, 2010: Final release of Common Core State Standards with approval of the Validation Committee
- (Note: These are NOT federal standards: they are a state-level coordinated effort led by National Governors Association-NGA and the Council of Chief State School Officers-CCSSO.)



As of June 2012, 45 states and D.C. had adopted CCSS

Source: http://www.ascd.org/public-policy/common-core-standards.aspx

Criteria for the Standards

- Fewer, clearer, and higher standards
- Aligned with college and work expectations
- Included rigorous content and application of knowledge through high-order skills
- Built upon strengths and lessons of current state standards
- Internationally benchmarked, so that all students are prepared to succeed in our global economy and society
- Based on evidence and research

CCSSI 2010; www.corestandards.org

Comparison of CCSS-M with Composite Profile of the Top-Achieving Countries

Topic	Grade 1	2	3	4	5	6	7	8
Whole Number Meaning	●	●	●	●	●			
Whole Number Operations	●	●	●	●	●			
Properties of Whole Numbers Operations			●	●	●	●		
Fractions		●	●	●	●	●		
Measurement Units	●	●	●	●	●	●	●	●
Polygons & Circles		●	●	●	●	●	●	●
Data Representation & Analysis	●	●	●	●	●	●	●	●
3-D Geometry					●	●	●	●
Measurement Estimation & Errors					●		●	
Number Theory				●		●	●	●
2-D Geometry Basics		●		●	●	●	●	●
Rounding & Significant Figures			●	●	●		●	
Relation of Decimals & Fractions			●	●	●	●		
Estimating Computations	●	●	●	●	●	●	●	
Perimeter, Area & Volume			●	●	●	●	●	●
Equations & Formulas		●	●	●	●	●	●	●
Decimals				●	●	●		
Patterns, Relations & Functions							●	●
Geometric Transformations						●	●	●
Properties of Decimals & Fractions				●	●	●		
Orders of Magnitude							●	●
2-D Coordinate Geometry					●	●	●	●
Exponents, Roots & Radicals							●	●
Percentages					●	●	●	
Negative Numbers, Integers & Their Properties						●	●	
Proportionality Concepts					●	●	●	●
Proportionality Problems					●	●	●	●
Rational Numbers & Their Properties							●	●
Constructions Using Straightedge & Compass							●	●
Systematic Counting								
Uncertainty & Probability								
Real Numbers & Their Properties								●
Congruence & Similarity								●
Slope								●
Validation & Justification								●
Estimating Quantity & Size				●	●			

● Intended in the Simulated Possible A+ Set of Standards Defining Complete Topic Coverage

Topic Intended in CCSSM

W. Schmidt, 2012, Michigan State University

This is NOT Business-as-Usual

1. More demanding standards and new topics
2. New assessments measuring more complex reasoning
3. Urgency to address performance gaps
4. Expanded data and new technologies transform instruction

Significance of Common Core

- Internationally benchmarked
- Structured for learning trajectories
- Supports student mobility from state-to-state
- Produces economies of scale
- Targets equity and customization
- Prepares for digital learning with real-time data

Necessary but NOT sufficient

- Cross Walks:
  - Necessary: Tell you what is new or different,
  - **But <u>not sufficient</u>**: Need to know *how areas and strands are restructured*
- Standard by standard attention:
  - Necessary: To understand the standards, indicate central content
  - **But <u>not sufficient</u>**: Does not emphasize *how concepts develop over time*

1. More demanding standards and new topics

A. Eight Mathematical Practices

B. Changes in Content and Grade Expectations (K-5, 6-8, 9-12): Earlier (or Later), and More Demanding

C. Structure and Learning Trajectories/Progressions

D. Interdisciplinary Content: Reading and Writing in Science and Technical Subjects

A. Eight Mathematical Practices

(CCSS-Math, 2010)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

A. Eight Mathematical Practices

- Practices are always interrelated with content. They must be addressed in tandem.
- Practices may provide a way to observe classrooms to see how the content is made understandable, challenging and engaging to students.

B. Changes in Content and Grade Expectations

K-5 Domains

Domains	Grade Level
Counting and Cardinality	K only
Operations and Algebraic Thinking	1-5
Number and Operations in Base Ten	1-5
Number and Operations--Fractions	3-5
Measurement and Data	1-5
Geometry	1-5

B. Changes: Middle Grades Domains

Domains	Grade Level
Ratio and Proportional Relationships	6-7
The Number System	6-8
Expressions and Equations	6-8
Functions	8
Geometry	6-8
Statistics and Probability	6-8

B. Changes: High School Conceptual Categories and Domains

- NUMBER AND QUANTITY
  - The Real Number System
  - Quantities
  - The Complex Number System
  - Vector and Matrix Quantities
- ALGEBRA
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations
  - Reasoning with Equations and Inequalities

B. Changes: High School Conceptual Categories and Domains

- FUNCTIONS OVERVIEW
  - Interpreting Functions
  - Building Functions
  - Linear, Quadratic and Exponential Models
  - Trigonometric Functions
- MODELING

B. Changes: High School Conceptual Categories and Domains

- GEOMETRY
  - Congruence
  - Similarity, Right Triangles and Trigonometry
  - Circles
  - Expressing Geometric Properties with Equations
  - Geometric Measurement and Dimension
  - Modeling with Geometry
- STATISTICS AND PROBABILITY
  - Interpreting Categorical and Quantitative Data
  - Making Inferences and Justifying Conclusions
  - Conditional Probability and the Rules of Probability
  - Using Probability to Make Decisions

B. Changes by Grade Bands: grades K-5

- Numeration and operation intensified, and introduced earlier
  - Early place value foundations in grade K
  - Regrouping as composing / decomposing in grade 2
  - Decimals to hundredths in grade 4
- All three types of measurement simultaneously
  - Non-standard, English and Metric
- Emphasis on fractions as numbers
- Emphasis on number line as visualization / structure

B. Changes by Grade Bands: grades 6-8

- Ratio and Proportion focused on in grade 6
  - Ratio, unit rates, converting measurement, tables of values, graphing, missing value problems
- Percents introduced in grade 6
- Statistics introduced in grade 6
  - Statistical variability (measures of central tendency, distributions, interquartile range, mean and absolute deviation, data shape)
- Rational numbers in grade 7
- Grade 8: One-third of high-school algebra for all students

B. Changes by Grade Bands: grades 6-8

- Now: much higher expectations at middle grades, where, collectively, our capacity, and our student performance, are weakest.
- Also--Much more pressure on elementary school teachers to “get the job done.”

B. Changes by Grade Bands: grades 9-12

- Supports *both/either* (a) continuing an integrated approach or (b) a traditional siloed approach (Algebra I, Geometry, Algebra II)--or new models that synthesize these two.

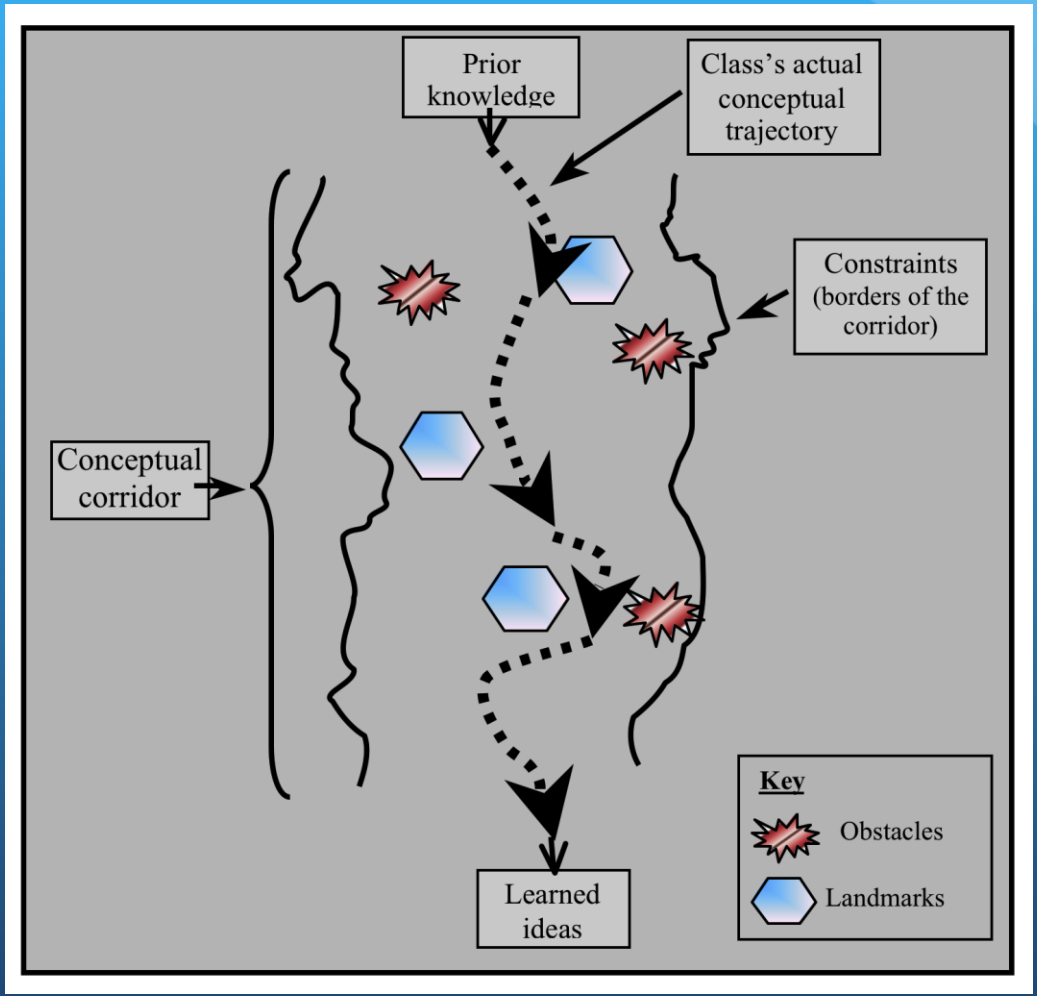
- All students must master some topics traditionally from algebra 2, or beyond
  - Simple periodic functions
  - Polynomials,
  - Radicals
  - More probability and statistics (correlation, random processes)
  - Introduction to mathematical modeling

C. Structure and Learning Progressions

"One promise of common state standards is that over time, they will allow research on learning progressions to inform and improve the design of Standards to a much greater extent than is possible today."

CCSS 2010, p.5

C. Learning Trajectory within a Conceptual Corridor



Confrey (2006) Design Studies Chapter
from the Cambridge Handbook of the Learning Sciences

C. A learning trajectory/progression is:

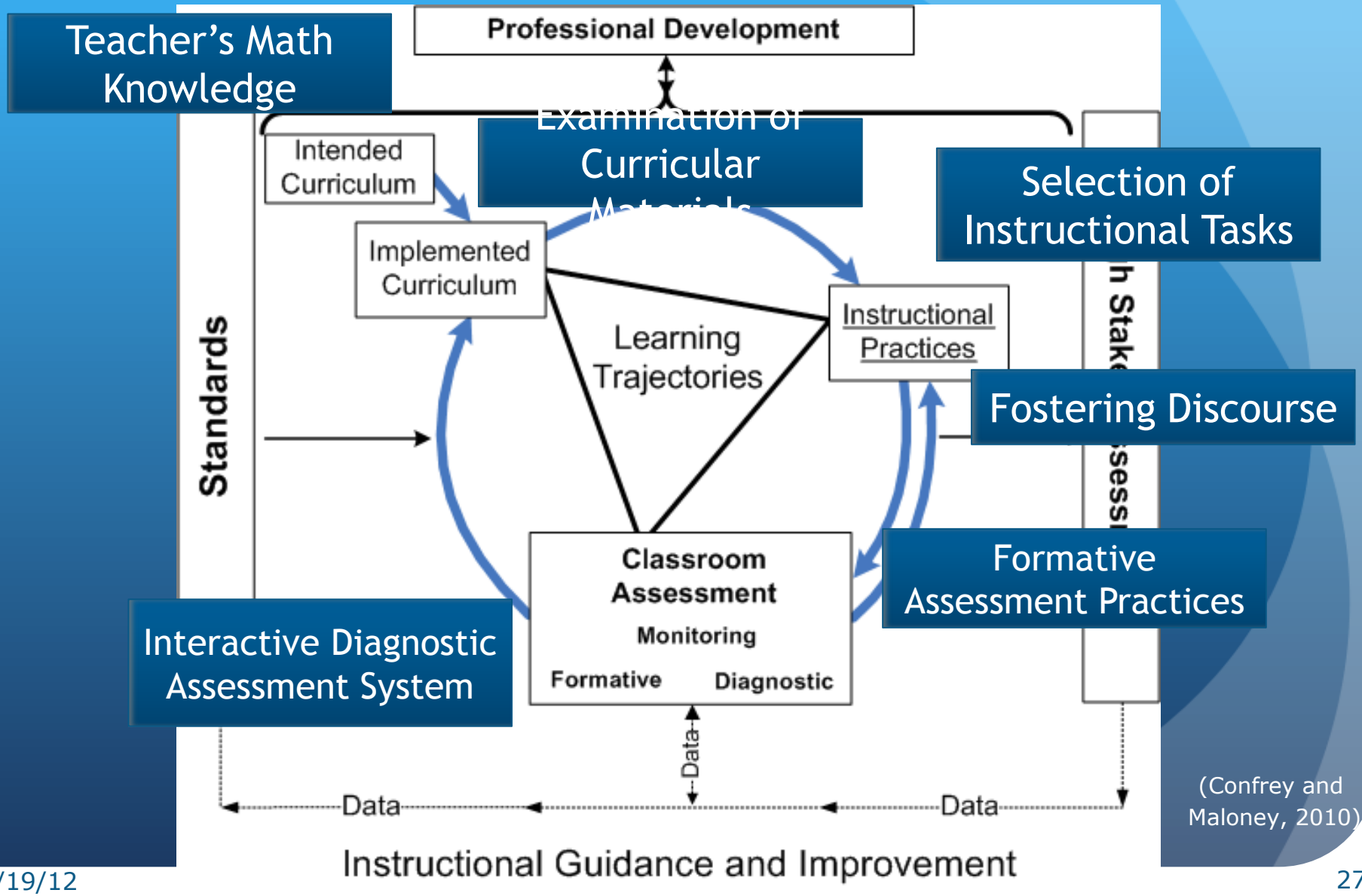
…a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time

(Confrey et al., 2009)

C. Value of Learning Trajectories to Teachers

- Know what to expect about students' preparation
- Know what teachers in the next grade expect of your students.
- More readily manage the range of preparation and needs of students in your class
- Identify clusters of related concepts at grade level
- Support student thinking and discourse to focus on conceptual development
- Engage in rich uses of classroom assessment

C. Learning Trajectories as Boundary Objects



(Confrey and Maloney, 2010)

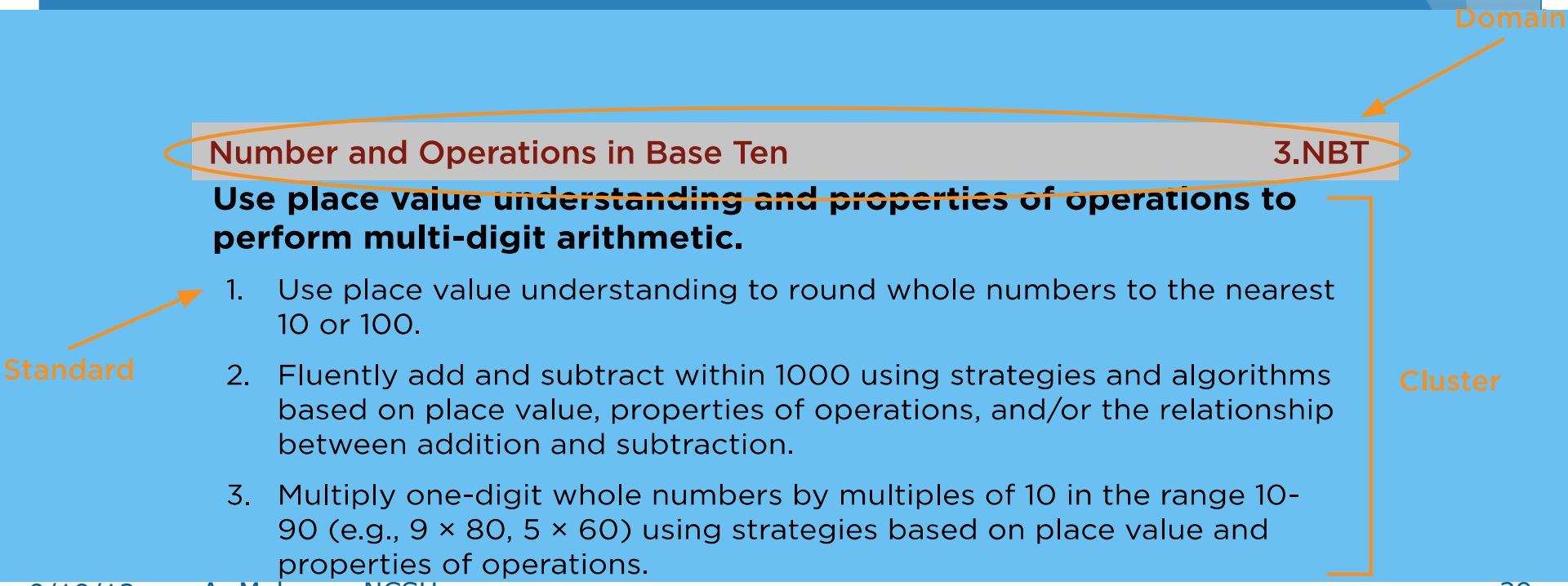
C. Structure

- [1.] Grade
  - [2.] Domains: larger groups of related standards. Standards from different domains may sometimes be closely related
    - [3.] Clusters of groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
      - [4.] Individual standards that define what students should understand and be able to do.

CCSSO, 2010

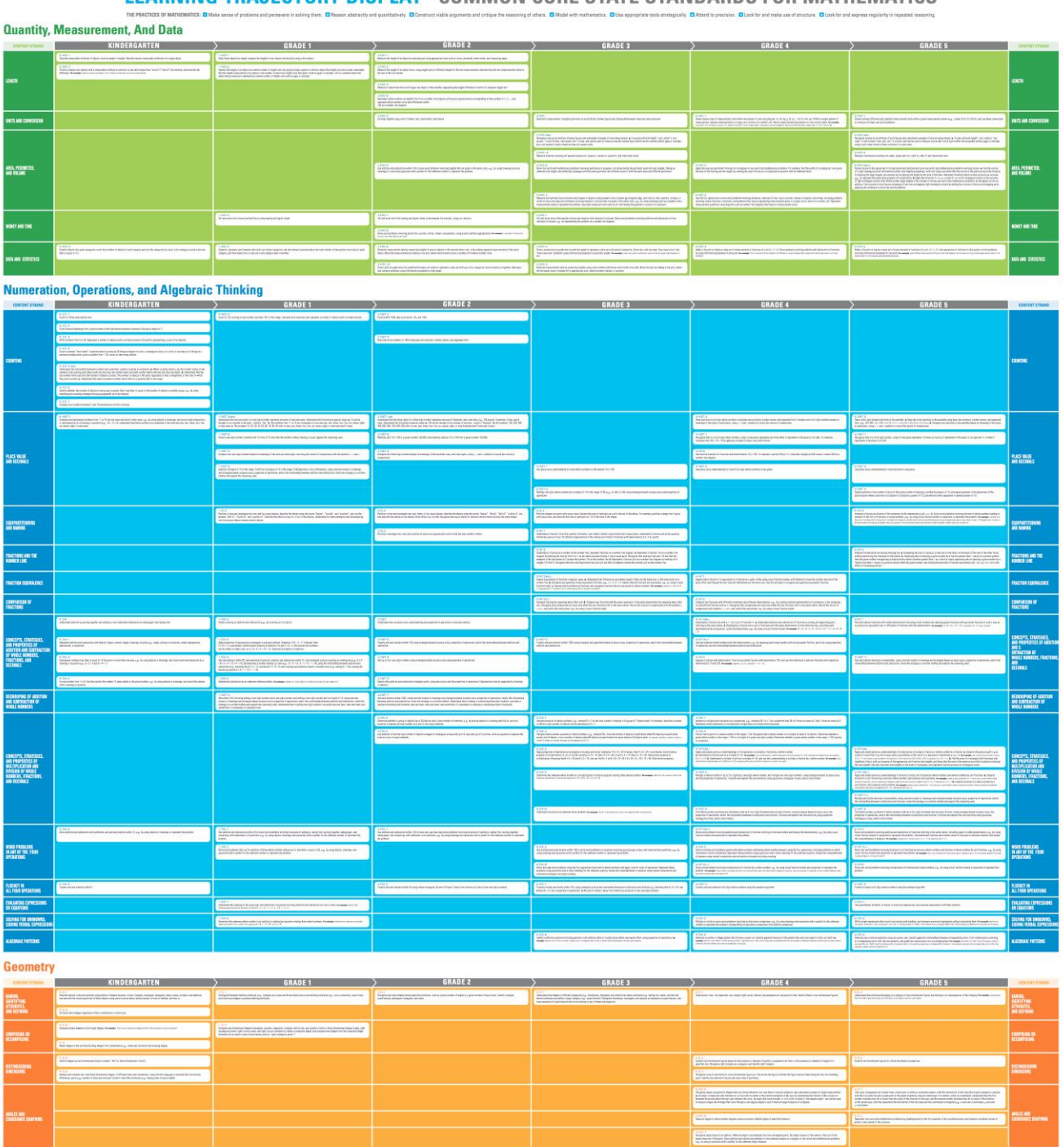
Design and Organization

- *Content standards* define what students should understand and be able to do
- *Clusters* are groups of related standards
- *Domains* are larger groups that progress across grades
- *Fewer, clearer, higher…*



Learning Trajectories View of the Common Core Standards (version 1: charts)

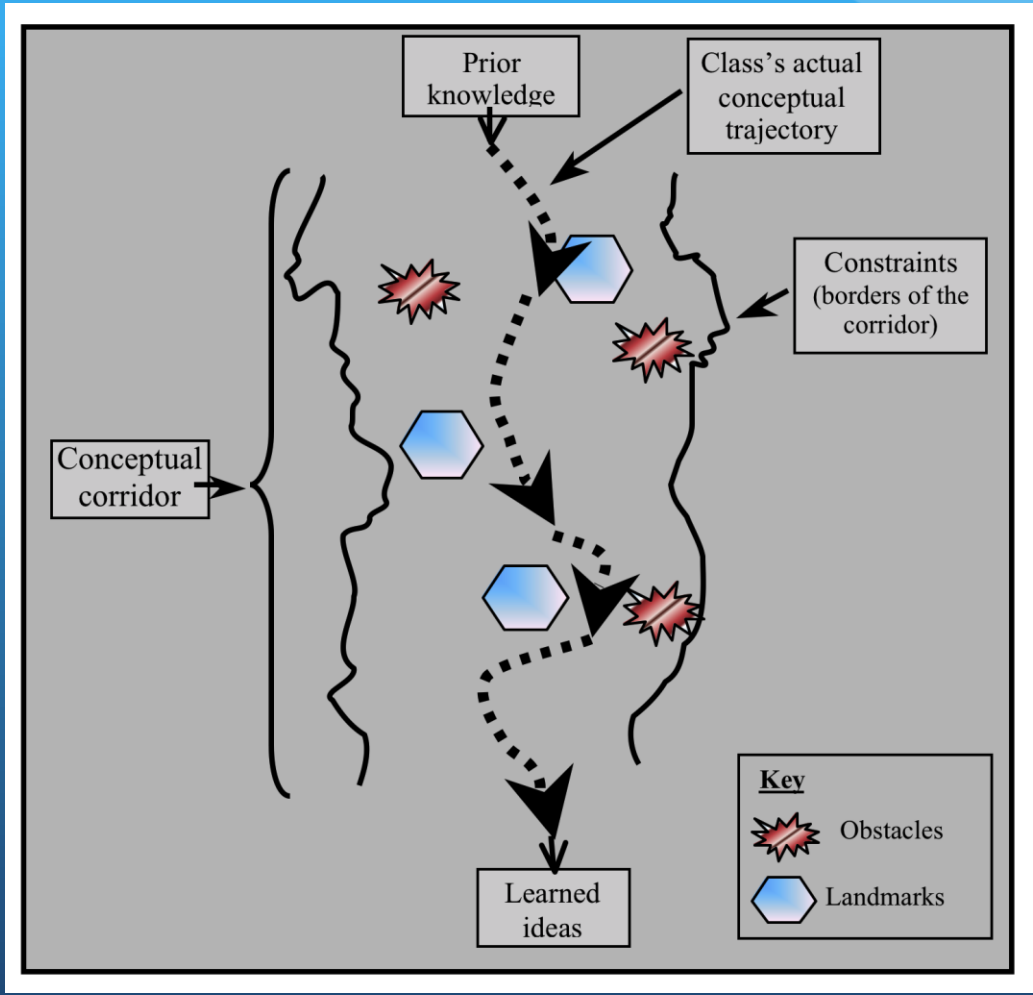
Content Strand	Kindergarten	Grade 1
Place Value and Decimals	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. [K.NBT.1] [K.NBT.1]	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones - called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). [1.NBT.2] [1.NBT.2]
		Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. [1.NBT.5] [1.NBT.5]
		Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. [1.NBT.3] [1.NBT.3]
		Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [1.NBT.6] [1.NBT.6]


LEARNING TRAJECTORY DISPLAY COMMON CORE STATE STANDARDS FOR MATHEMATICS
THE PRACTICES OF MATHEMATICS: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.
Quantity, Measurement, And Data
KINDERGARTEN
GRADE 1
GRADE 2
GRADE 3
GRADE 4
GRADE 5
LENGTH
UNITS AND CONVERSION
AREA, PERIMETER, AND VOLUME
MONEY AND TIME
DATA AND STATISTICS
Numeration, Operations, and Algebraic Thinking
KINDERGARTEN
GRADE 1
GRADE 2
GRADE 3
GRADE 4
GRADE 5
COUNTING
PLACE VALUE AND DECIMALS
EQUIPARTITIONING AND NAMING
FRACTIONS AND THE NUMBER LINE
FRACTION EQUIVALENCE
COMPARISON OF FRACTIONS
CONCEPTS, STRATEGIES, AND PROPERTIES OF ADDITION AND SUBTRACTION OF WHOLE NUMBERS, FRACTIONS, AND DECIMALS
REGROUPING OF ADDITION AND SUBTRACTION OF WHOLE NUMBERS
CONCEPTS, STRATEGIES, AND PROPERTIES OF MULTIPLICATION AND DIVISION OF WHOLE NUMBERS, FRACTIONS, AND DECIMALS
WORD PROBLEMS OF ANY OF THE FOUR OPERATIONS
FLUENCY IN ALL FOUR OPERATIONS
EVALUATING EXPRESSIONS OR EQUATIONS
SOLVING FOR UNKNOWNS, GIVING VERBAL EXPRESSIONS
ALGEBRAIC PATTERNS
Geometry
KINDERGARTEN
GRADE 1
GRADE 2
GRADE 3
GRADE 4
GRADE 5
NAMING, IDENTIFYING, AND DESCRIBING
COMPOSING OR DECOMPOSING
DISTINGUISHING DIMENSIONS
ANGLES AND COORDINATE GRAPHING

Part 1 Take-Away Messages

- Common Core Standards represent an opportunity and a challenge. They are a legacy to young teachers.
- New assessments will test more complex reasoning.
- Middle grades will be critical to student success.
- Elementary grades instruction critical to [--everything--] middle grades preparation and success.
- Implementation must consider naming, phasing, sequencing, obligatory and ongoing professional development, and public relations.
- Specificity of standards should increase equity, if students are provided adequate opportunities to learn
- Central hosting, technology services models, and diversity of content offers enormous opportunities and pitfalls.

Part 2: Digging In with Learning Trajectories



Confrey (2006) Design Studies Chapter
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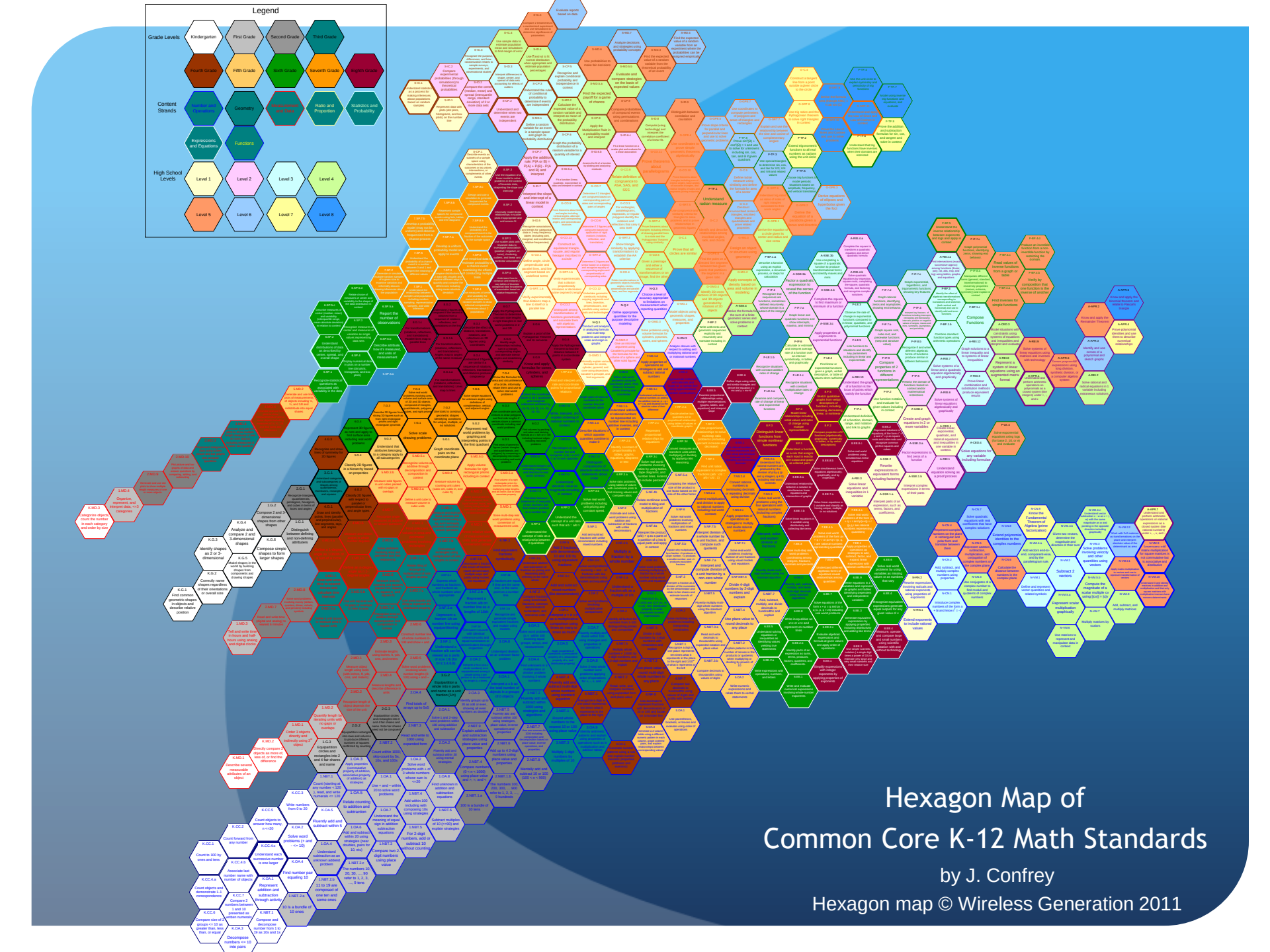
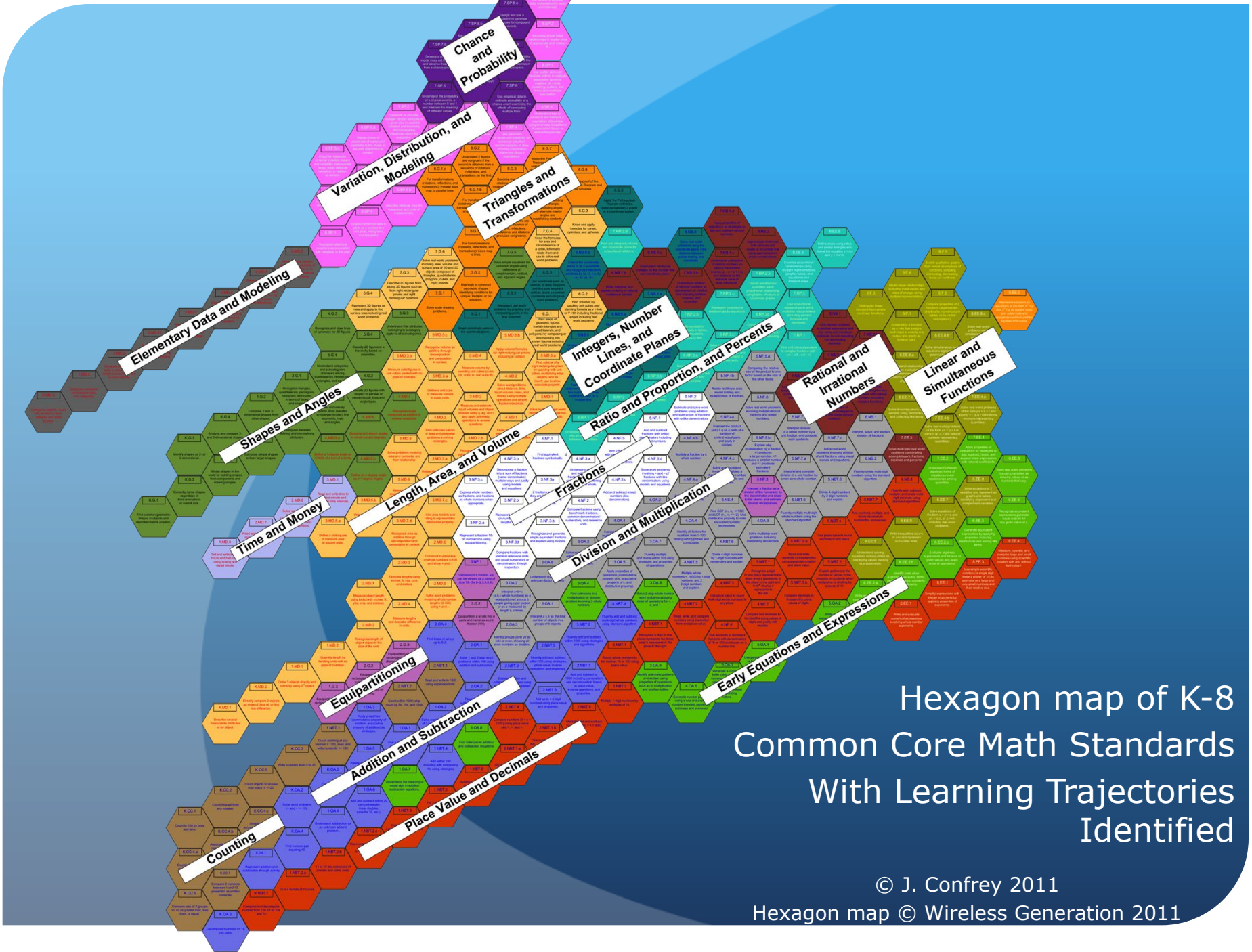
Complementary Approaches to Unpacking the CCSS-M

- Go to: www.turnonccmath.net


Legend
Grade Levels
Kindergarten
First Grade
Second Grade
Third Grade
Fourth Grade
Fifth Grade
Sixth Grade
Seventh Grade
Eighth Grade
Content Strands
Number and Operations
Geometry
Measurement and Data
Ratio and Proportion
Statistics and Probability
Expressions and Equations
Functions
High School Levels
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7
Level 8
Hexagon Map of
Common Core K-12 Math Standards
by J. Confrey
Hexagon map © Wireless Generation 2011



Hexagon map of K-8 Common Core Math Standards With Learning Trajectories Identified

List of 18 K-8 Learning Trajectories

- Counting
- Place Value and Decimals
- Addition and Subtraction
- Equipartitioning
- Time and Money
- Length, Area and Volume
- Fractions
- Multiplication and Division
- Ratio and Proportion, and Percent

List of 18 K-8 Learning Trajectories

- Shapes and Angles
- Triangles and Transformations
- Elementary Data and Modeling
- Variation, Distribution and Modeling
- Chance and Probability
- Integers, Number lines, and Coordinate Planes
- Rational and Irrational Numbers
- Early Equations and Expressions
- Linear and Simultaneous Functions

Complementary Approaches to Unpacking the CCSS-M

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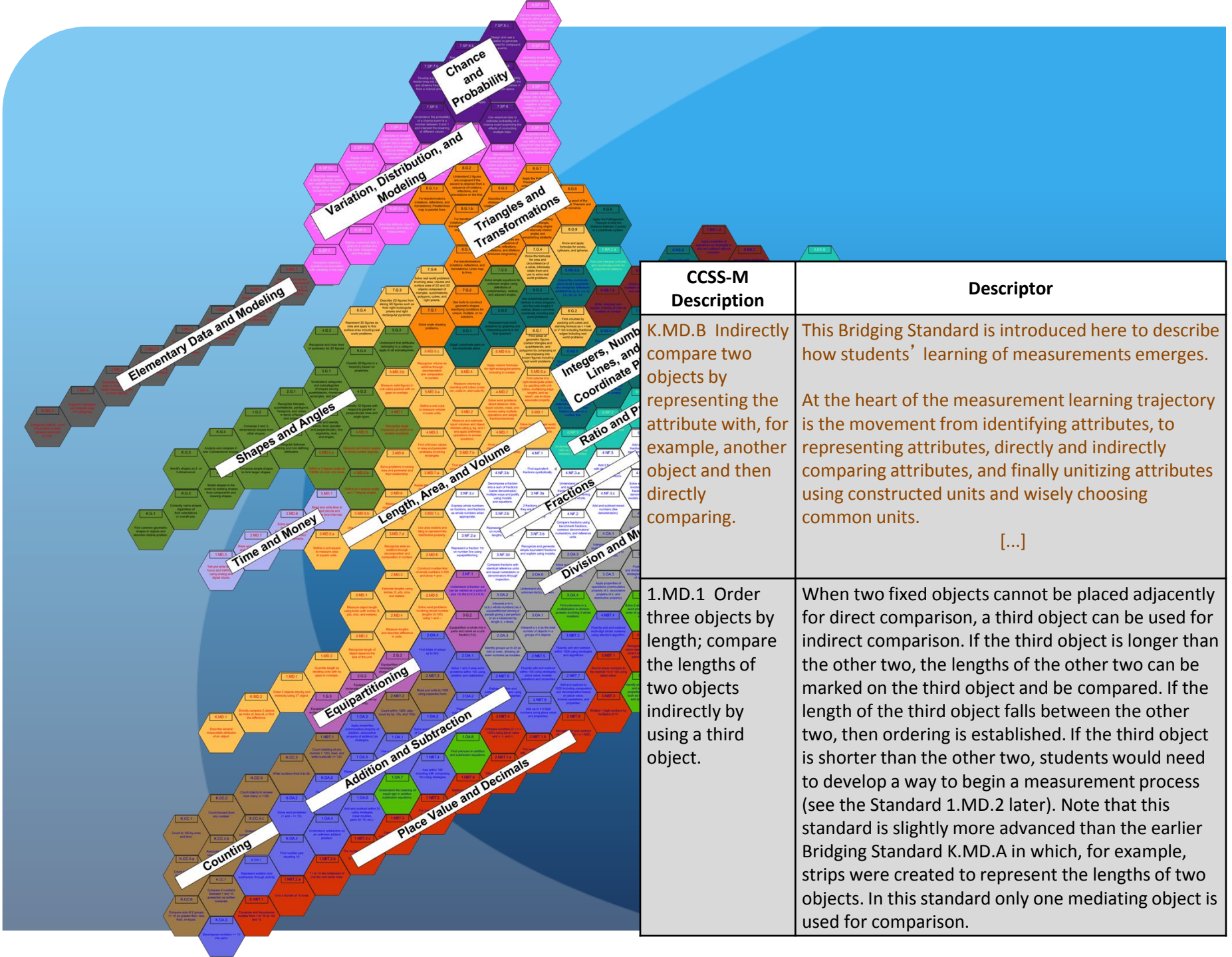
1. Student strategies, representations, and misconceptions
2. Underlying cognitive and conceptual principles
3. Mathematical distinctions and multiple models
4. Coherent structure
5. Bridging standards



CCSS-M Description	Descriptor
K.MD.B Indirectly compare two objects by representing the attribute with, for example, another object and then directly comparing.	This Bridging Standard is introduced here to describe how students' learning of measurements emerges. At the heart of the measurement learning trajectory is the movement from identifying attributes, to representing attributes, directly and indirectly comparing attributes, and finally unitizing attributes using constructed units and wisely choosing common units. [...]
1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.	When two fixed objects cannot be placed adjacently for direct comparison, a third object can be used for indirect comparison. If the third object is longer than the other two, the lengths of the other two can be marked on the third object and be compared. If the length of the third object falls between the other two, then ordering is established. If the third object is shorter than the other two, students would need to develop a way to begin a measurement process (see the Standard 1.MD.2 later). Note that this standard is slightly more advanced than the earlier Bridging Standard K.MD.A in which, for example, strips were created to represent the lengths of two objects. In this standard only one mediating object is used for comparison.

- End